Day 38 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Binomial probabilities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X ~ B(18, 0.35). Find, to 3 significant figures:
(a) P(8 ≤ X ≤ 9) (b) P(X > 8)

(a) $P(X \leq 9) - P(X \leq 7) = 0.940 - 0.728 = 0.212$
(b) $1 - P(X \leq 8) = 1 - 0.861 = 0.139$
Answer: (a) 0.212 (b) 0.139

2 Arc length and sector area (All boards)

A sector of a circle of radius 10 cm has angle 210°.
Find, in terms of π, the arc length, the area and the perimeter of the sector.

$\theta = 210° = \frac{7\pi}{6}$ radians

Arc length $= r\theta = 10 \times \frac{7\pi}{6} = \frac{35\pi}{3}$ cm

Area $= \frac{1}{2}r^2\theta = \frac{1}{2} \times 10^2 \times \frac{7\pi}{6} = \frac{175\pi}{3}$ cm^2

Perimeter $= 2r + \text{arc} = \frac{35\pi + 60}{3}$ cm

Answer: arc $\frac{35\pi}{3}$ cm, area $\frac{175\pi}{3}$ cm^2, perimeter $\frac{35\pi + 60}{3}$ cm

3 Three-set Venn diagram counts (All boards)

Of 33 students, 14 study Maths (M), 17 Physics (P) and 13 Chemistry (C).
7 study M and P, 5 study M and C, 4 study P and C, and 1 study all three.
One student is chosen at random. Given that a student studies Chemistry, find the probability that they study all three.

Venn regions: M only 3, P only 7, C only 5,
MP only 6, MC only 4, PC only 3, all three 1, outside all 4
Studies Chemistry: 13
Studies all three: 1

Probability $= \frac{1}{13}$

Answer: $\frac{1}{13}$

4 Geometric sequences: finding a and r (All boards)

The 1st term of a geometric sequence is 3 and the 4th term is −81.
Find the common ratio and the first term.
Find the sum of the first 5 terms.

$r^3 = \frac{-81}{3} = -27$ so $r = -3$

The first term is $a = 3$

$S_5 = \frac{a(r^5 - 1)}{(r - 1)} = \frac{3((-243) - 1)}{-4}$

Answer: $r = -3$, $a = 3$, $S_5 = 183$

5 Constant acceleration in vector form (AQA, Edexcel, OCR, MEI, Eduqas)

A particle P has constant acceleration $(-\mathbf{i} + 3\mathbf{j})$ m s^{-2}. When t = 0, P is at $(2\mathbf{i} + 4\mathbf{j})$ m moving with velocity $(5\mathbf{i} - 5\mathbf{j})$ m s^{-1}.
Find (a) when P moves parallel to **j**, (b) its position vector then.

$\mathbf{v} = \mathbf{u} + \mathbf{a}t = (5\mathbf{i} - 5\mathbf{j}) + t(-\mathbf{i} + 3\mathbf{j})$
(a) Parallel to **j**: the **i** component is zero: $5 - t = 0$, so $t = 5$
(b) $\mathbf{r} = \mathbf{r}_0 + \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$
$= (2\mathbf{i} + 4\mathbf{j}) + 5(5\mathbf{i} - 5\mathbf{j}) + 12.5(-\mathbf{i} + 3\mathbf{j})$
$= 14.5\mathbf{i} + 16.5\mathbf{j}$
Answer: (a) $t = 5$ (b) $\mathbf{r} = (14.5\mathbf{i} + 16.5\mathbf{j})$ m

6 Differentiating sec, cosec, tan and cot (All boards)

y = tan(3x).

Find $\frac{dy}{dx}$ and its exact value at x = $\frac{\pi}{18}$.

$\frac{dy}{dx} = 3\sec^2(3x)$

At x = $\frac{\pi}{18}$ the angle 3x = $\frac{\pi}{6}$, so the gradient is 4

Answer: $\frac{dy}{dx} = 3\sec^2(3x)$; 4

7 Binomial expansion: (a + bx)^n for rational n (All boards)

Expand $(9 + 2x)^{-\frac{1}{2}}$ in ascending powers of x, up to and including x^2.
State the range of x for which the expansion is valid.

$(9 + 2x)^{-\frac{1}{2}} = 9^{-\frac{1}{2}}(1 + \frac{2x}{9})^{-\frac{1}{2}}$

$9^{-\frac{1}{2}} = \frac{1}{3}$

Expand $(1 + y)^{-\frac{1}{2}}$ with y = $\frac{2x}{9}$

Valid for $|\frac{2x}{9}| < 1$, i.e. $|x| < \frac{9}{2}$

Answer: $\frac{1}{3} - \frac{1}{27}x + \frac{1}{162}x^2$; $|x| < \frac{9}{2}$

8 Hinged rod held by a string (AQA, Edexcel, OCR, MEI)

A uniform rod AB (6 kg, 4 m) is smoothly hinged at A to a vertical wall, with a 4 kg particle at B. A light string from B to the wall above A holds the rod horizontal, at angle θ to it, tan θ = $\frac{4}{3}$. Find (a) the tension, (b) the magnitude of the hinge force. Take g = 9.8. Give answers to 3 s.f. if not exact.

Moments about A: T sin θ × 4 = 6 g × 2 + 4 g × 4
T sin θ = (3 + 4)g = 68.6, so (a) T = 85.8
Horizontal component of hinge force: X = T cos θ = 51.5
Vertical: Y + T sin θ = (6 + 4)g, so Y = 6 g ÷ 2 = 29.4
(b) Force = $\sqrt{(X^2 + Y^2)} = \sqrt{(51.5^2 + 29.4^2)}$
Answer: (a) 85.75 N (b) 59.3 N

9 Recurrence relations: unknown first term (All boards)

A sequence is defined by $u_1 = k$ and $u_{n+1} = 2u_n - 3$.

Given that $u_3 = -5$, find (a) the value of k, (b) $\sum_{r=1}^{4} u_r$.

$u_2 = 2k - 3$, $u_3 = 2(2k - 3) - 3$
$4k - 9 = -5$ so k = 1
Terms: 1, −1, −5, −13
Sum = −18
Answer: (a) k = 1 (b) −18

10 Optimisation problems (All boards)

A closed cylinder has radius r cm and volume 54π cm^3.

Show that its surface area is $A = 2\pi r^2 + \frac{108\pi}{r}$, and find the minimum value of A in terms of π.

$h = \frac{54}{r^2}$, so $A = 2\pi r^2 + 2\pi rh = 2\pi r^2 + \frac{108\pi}{r}$

$\frac{dA}{dr} = 4\pi r - \frac{108\pi}{r^2} = 0$, so $r^3 = 27$ and $r = 3$

$\frac{d^2A}{dr^2} = 4\pi + \frac{216\pi}{r^3} > 0$, so a minimum

$A = 2\pi \times 9 + \frac{108\pi}{3} = 54\pi$

Answer: A_min = 54π cm^2 (r = 3)