Day 37 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Foot of perpendicular and distance to a line (CIE (9709); Further Maths on other boards)

The line l has equation $\mathbf{r} = \begin{pmatrix} -7 \\ -6 \\ 4 \end{pmatrix} + t\begin{pmatrix} 2 \\ 3 \\ -2 \end{pmatrix}$.

Find the coordinates of the foot of the perpendicular from P(2, 1, −2) to l, and the exact distance from P to l.

Foot F = A + t d: $\mathrm{PF} = \begin{pmatrix} 2t - 9 \\ 3t - 7 \\ -2t + 6 \end{pmatrix}$

PF·d = 0 gives 17t − 51 = 0, so t = 3

F = (−1, 3, −2); $\mathrm{PF} = \begin{pmatrix} -3 \\ 2 \\ 0 \end{pmatrix}$, so $|\mathrm{PF}| = \sqrt{13}$

Answer: F(−1, 3, −2), distance $\sqrt{13}$

2 Arithmetic sequences: finding a and d (All boards)

The 5th term of an arithmetic sequence is −23 and the 8th term is −35.

Find the first term and common difference, and the sum of the first 12 terms.

$u_8 - u_5 = 3\,d = -35 - (-23)$ so $d = -4$

$a + 4\,d = -23$ so $a = -7$

$S_{12} = \frac{12}{2}(2 \times (-7) + 11 \times (-4))$

Answer: a = −7, d = −4, S_{12} = −348

3 Binomial probabilities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X ~ B(13, 0.1). Find, to 3 significant figures:

(a) P(4 ≤ X ≤ 6) (b) P(X > 4)

(a) P(X ≤ 6) − P(X ≤ 3) = 1.00 − 0.966 = 0.0341

(b) 1 − P(X ≤ 4) = 1 − 0.994 = 0.00646

Answer: (a) 0.0341 (b) 0.00646

4 Trapezium rule with over or under estimate (AQA, Edexcel, OCR, MEI, Eduqas)

Use the trapezium rule with 4 strips to estimate $\int_0^2 2^x\,dx$, giving your answer to 3 s.f.

State, with a reason, whether your estimate is an overestimate or an underestimate.

h = $\frac{1}{2}$; y-values at x = 0, $\frac{1}{2}$, 1, $\frac{3}{2}$, 2: 1.0000, 1.4142, 2.0000, 2.8284, 4.0000

$T = \frac{1}{4}[1.0000 + 4.0000 + 2(1.4142 + 2.0000 + 2.8284)]$

T = 4.37

The curve is convex (y'' > 0) on this interval, so the chords lie above the curve: overestimate

Answer: 4.37, an overestimate

5 Coding mean and standard deviation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

For 10 values of x, $\Sigma(x - 200) = 10$ and $\Sigma(x - 200)^2 = 20$.

Find the mean and standard deviation of x.

Let y = x − 200: $\bar{y} = \frac{10}{10} = 1$

Mean of x = 1 + 200 = 201

Variance = $\frac{20}{10} - 1^2 = 1$

Subtracting a constant does not change spread, so sd = 1

Answer: mean 201, sd 1

6 Position vector and velocity (AQA, Edexcel, OCR, MEI, Eduqas)

A particle P has position vector $\mathbf{r} = (t^3 - 3t)\mathbf{i} + (4t^2 - 8t)\mathbf{j}$ metres at time t s. Find (a) the speed of P when t = 4, (b) the value of t when P is moving parallel to $\mathbf{i}$.

$\mathbf{v} = d\mathbf{r}/dt = (3t^2 - 3)\mathbf{i} + (8t - 8)\mathbf{j}$

(a) t = 4: $\mathbf{v} = 45\mathbf{i} + 24\mathbf{j}$

Speed = $\sqrt{(45^2 + 24^2)} = \sqrt{2601}$

(b) Parallel to $\mathbf{i}$ means the $\mathbf{j}$ component is zero: $8t - 8 = 0$, so t = 1

Answer: (a) 51 m s^{-1} (b) t = 1

7 Product rule: stationary points (All boards)

$y = x^3 \ln x$ for $x > 0$.

Find the exact coordinates of the stationary point and show that it is a minimum.

$\frac{dy}{dx} = 3x^2 \ln x + x^3 \times \frac{1}{x} = x^2(3 \ln x + 1)$

$\frac{dy}{dx} = 0$, $x > 0$: $\ln x = -\frac{1}{3}$, so $x = e^{-\frac{1}{3}}$

$y = x^3 \ln x = e^{-1} \times (-\frac{1}{3}) = -\frac{e^{-1}}{3}$

$\frac{d^2y}{dx^2} = 3e^{-\frac{1}{3}} > 0$ at this point, so it is a minimum

Answer: $(e^{-\frac{1}{3}}, -\frac{e^{-1}}{3})$, minimum

8 Circle: tangent at a point (AQA, Edexcel, OCR, MEI, Eduqas)

The circle $x^2 + y^2 + 4x + 2y - 164 = 0$ passes through P(3, −13).

(a) Find the equation of the tangent at P, in the form ax + by = c.

(b) Find the coordinates of the point where this tangent crosses the x-axis.

Centre (−2, −1). Gradient of radius CP = $-\frac{12}{5}$

Gradient of tangent = $\frac{5}{12}$

$y + 13 = \frac{5}{12}(x - 3)$: $5x - 12y = 171$

y = 0: $x = \frac{171}{5}$

Answer: (a) 5x − 12y = 171 (b) $(\frac{171}{5}, 0)$

9 Projectile: perpendicular directions (AQA, Edexcel, OCR, MEI, Eduqas)

A ball is projected from the ground at 32 m s^{-1} at angle α above the horizontal, $\tan\alpha = \frac{15}{8}$. At time T s, while in flight, it is moving perpendicular to its initial direction. Find (a) T, (b) the height of the ball then. Take g = 9.8. Give answers to 3 s.f. if not exact.

Initial velocity $(\frac{256}{17}, \frac{480}{17})$. At time T the velocity is $(\frac{256}{17}, \frac{480}{17} - 9.8T)$.

Perpendicular: scalar product = 0: $(\frac{256}{17})^2 + \frac{480}{17}(\frac{480}{17} - 9.8T) = 0$

$32^2 - 9.8T \times \frac{480}{17} = 0$ (using $\sin^2\alpha + \cos^2\alpha = 1$)

(a) $T = 32 \div (9.8 \times \frac{15}{17}) = \frac{544}{147}$

(b) $h = \frac{480}{17} \times \frac{544}{147} - 4.9 \times \frac{544}{147}^2 = \frac{11776}{315}$

Answer: (a) T = 3.70 s (b) 37.4 m

10 Periodic sequences (All boards)

A sequence is defined by $u_1 = \frac{1}{2}$ and $u_{n+1} = -\frac{1}{u_n}$.

(a) Find u_2 and u_3.

(b) Find u_{99} and $\sum_{r=1}^{99} u_r$.

$u_2 = -2$, $u_3 = \frac{1}{2}$, $u_4 = -2$

$u_3 = u_1$, so the sequence repeats with period 2.

$99 = 2 \times 49 + 1$, so $u_{99} = u_1 = \frac{1}{2}$

Sum of one period = $-\frac{3}{2}$; 49 full periods plus 1 extra term(s)

Answer: (a) -2, $\frac{1}{2}$ (b) $u_{99} = \frac{1}{2}$, sum = -73