Day 364 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Trigonometric equations (quadratic in sin or cos) (All boards)

Solve $2\cos^2 x - \sin x + 1 = 0$
for $0° \leq x \leq 180°$.

Use $\cos^2 x \equiv 1 - \sin^2 x$: $2\sin^2 x + \sin x - 3 = 0$

Solving the quadratic: $\sin x = 1$ or $\sin x = -\frac{3}{2}$

$\sin x = 1$: $x = 90°$

$\sin x = -\frac{3}{2}$ has no solutions ($|\sin x| \leq 1$)

Answer: $x = 90°$

2 Partial fractions: distinct linear factors (All boards)

Express $\frac{-7x - 7}{(x - 2)(x + 5)}$ in partial fractions.

Let $\frac{-7x - 7}{(x - 2)(x + 5)} \equiv \frac{A}{(x - 2)} + \frac{B}{(x + 5)}$
$-7x - 7 \equiv A(x + 5) + B(x - 2)$
Substitute $x = 2$: $A = -3$
Substitute $x = -5$: $B = -4$
Answer: $\mathbf{\frac{-3}{(x - 2)} + \frac{-4}{(x + 5)}}$

3 Smallest whole number solving an exponential inequality (All boards)

Find the smallest positive integer n such that
$(\frac{9}{10})^n < 0.02$.

Take logs: $n \ln \frac{9}{10} < \ln 0.02$

$\ln \frac{9}{10}$ is negative, so dividing reverses the inequality:

$n > \frac{\ln 0.02}{\ln \frac{9}{10}} = 37.13$, so $n = 38$

Answer: $n = 38$

4 Passing a point twice (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A stone is projected vertically upwards from the ground. It passes a ledge 3 s after projection and again 7 s after projection. Find (a) the speed of projection, (b) the height of the ledge, (c) the greatest height. Take $g = 9.8$.

Same height at $t = 3$ and $t = 7$, so the stone is at its highest point at $t = 5$
(a) At the top $v = 0$: $0 = u - 9.8 \times 5$, so $u = 49$
(b) $s = ut - 4.9t^2$ with $t = 3$: $s = 49 \times 3 - 4.9 \times 3^2 = 102.9$
(c) $v^2 = u^2 - 2gs$: $0 = 49^2 - 19.6s$, $s = 122.5$
Answer: (a) 49 m s^{-1} (b) 102.9 m (c) 122.5 m

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5 Conditional probability without replacement (All boards)

A bag has 7 green and 5 yellow counters. Two are taken at random without replacement. Given that the counters are the same colour, find the probability they are both green.

P(both green) = $\frac{7}{12} \times \frac{6}{11} = \frac{7}{22}$

P(both yellow) = $\frac{5}{12} \times \frac{4}{11} = \frac{5}{33}$

P(same colour) = $\frac{31}{66}$

P(both green | same) = $\frac{\frac{7}{22}}{\frac{31}{66}}$

= $\frac{21}{31}$

Answer: $\mathbf{\frac{21}{31}}$

6 Integration by substitution (All boards)

Use the substitution u = x + 3 to find the exact value of

$\int_0^{13} \frac{x}{\sqrt{x+3}}\,dx$.

x = u − 3, du = dx; limits u = 3 to u = 16

$I = \int_3^{16} \frac{u-3}{\sqrt{u}}\,du = \int_3^{16} (u^{\frac{1}{2}} - 3u^{-\frac{1}{2}})\,du$

$= [\frac{2}{3}u^{\frac{3}{2}} - 6u^{\frac{1}{2}}]$

$= \frac{12\sqrt{3}+56}{3}$

Answer: $\mathbf{\frac{12\sqrt{3}+56}{3}}$

7 Equation of a reciprocal curve from its graph (All boards)

The diagram shows the curve $y = \frac{k}{x-a} + b$ with asymptotes as shown. Find the values of k, a and b, and the coordinates where the curve crosses the axes.

Asymptotes x = 1, y = 3 so a = 1, b = 3

Through (5, 4): $4 = \frac{k}{4} + 3$

k = (4 − 3)(4) = 4

x = 0: y = −1; y = 0: $x = -\frac{1}{3}$

Answer: k = 4, a = 1, b = 3; (0, −1) and $(-\frac{1}{3}, 0)$

8 Velocity, rest and total distance (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle P moves on the x-axis with velocity $v = (6t^2 - 30t + 36)$ m s^{-1} at time t s, $t \geq 0$, and starts at O. Find (a) the times when P is at instantaneous rest, (b) the total distance P travels in the first 5 s.

(a) v = 6(t − 2)(t − 3) = 0, so t = 2 or t = 3

(b) Displacement $x = \int v\,dt = 2t^3 - 15t^2 + 36t$

x(2) = 28, x(3) = 27, x(5) = 55

Direction changes at t = 2 and t = 3, so distance = |28| + |27 − 28| + |55 − 27|

Answer: (a) t = 2, t = 3 (b) 57 m

9 Tangent conditions (All boards)

Find the set of values of k for which the line $y = 4x + k$ does not meet the curve $y = 2x^2 + 4x - 3$.

$2x^2 + 4x - 3 = 4x + k$ gives $2x^2 + (-3 - k) = 0$

Discriminant: $0^2 - 4 \times 2(-3 - k) = 24 + 8k$

No intersection: discriminant < 0

Answer: $k < -3$

10 Bayes' theorem in context (All boards)

0.5% of people have a disease. The screening test is positive for 98% of those who have it and for 2% of those who do not.

(a) Find the probability that a person tests positive.

(b) Given a positive test, find the probability that the person has the disease (3 s.f.).

(a) $P(+) = 0.005 \times 0.98 + 0.995 \times 0.02$

$= 0.0049 + 0.0199 = 0.0248$

(b) $P(\text{has it} \mid +) = \dfrac{0.005 \times 0.98}{0.0248}$

$= 0.198$

Answer: (a) 0.0248 (b) 0.198