Day 363 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Equation of a cubic from its graph (All boards)

The diagram shows a sketch of the curve y = f(x), where f(x) is a cubic.
Find f(x), giving your answer in expanded form.

Roots at $x = -3, 1, -2$, so $f(x) = k(x + 3)(x - 1)(x + 2)$
$f(0) = -12$: $k(3)(-1)(2) = -12$
$k = 2$, then expand
Answer: $f(x) = 2x^3 + 8x^2 + 2x - 12$

2 Equilibrium on a slope: range of force (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A 6 kg particle rests on a rough plane inclined at α, $\tan\alpha = \frac{3}{4}$, $\mu = 0.5$. A force of P N acts up a line of greatest slope and the particle stays at rest. Find the range of values of P. Take g = 9.8.

$\sin\alpha = 0.6$, $\cos\alpha = 0.8$, $R = 6g\cos\alpha = 47.04$ N
Limiting friction $= \mu R = 23.52$ N
P smallest: friction acts up the slope: $P + \mu R = 6g\sin\alpha$, $P = 35.28 - 23.52$
P greatest: friction acts down the slope: $P = 35.28 + 23.52$
Answer: $11.76 \text{ N} \leq P \leq 58.8 \text{ N}$

3 Area and perimeter of a segment (exact) (All boards)

The circle has centre O and radius 8 cm. Angle AOB = $\frac{2\pi}{3}$.
Find the exact area and the exact perimeter of the shaded segment.

Sector area $= \frac{1}{2}r^2\theta = \frac{64\pi}{3}$; triangle OAB $= \frac{1}{2}r^2\sin\theta = 16\sqrt{3}$

Segment area = sector − triangle $= \frac{64\pi - 48\sqrt{3}}{3}$

Chord AB $= 2r\sin\frac{\theta}{2} = 8\sqrt{3}$; arc $= r\theta = \frac{16\pi}{3}$

Perimeter = arc + chord $= \frac{24\sqrt{3} + 16\pi}{3}$

Answer: area $= \frac{64\pi - 48\sqrt{3}}{3}$ cm^2, perimeter $= \frac{24\sqrt{3} + 16\pi}{3}$ cm

4 Coding mean and standard deviation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

For 20 values of x, $\Sigma(x - 50) = 30$ and $\Sigma(x - 50)^2 = 50$.
Find the mean and standard deviation of x.

Let $y = x - 50$: $\bar{y} = \frac{30}{20} = 1.5$
Mean of $x = 1.5 + 50 = 51.5$
Variance $= \frac{50}{20} - 1.5^2 = 0.25$
Subtracting a constant does not change spread, so sd = 0.5
Answer: mean 51.5, sd 0.5

5 Equation after a sequence of transformations (All boards)

The curve with equation $y = -x^2 + 2x + 4$ is transformed by a stretch, scale factor 2, parallel to the x-axis, followed by a reflection in the x-axis.
Find the equation of the image curve.

Stretch parallel to x: replace x by $\frac{x}{2}$: $y = -\frac{1}{4}x^2 + x + 4$

Reflect in x-axis: y becomes −y: $y = \frac{1}{4}x^2 - x - 4$

Answer: $y = \frac{1}{4}x^2 - x - 4$

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6 Small angle approximations: limits (AQA, Edexcel, OCR, MEI, Eduqas)

For small x (in radians), use the small angle approximations to find the value that $\frac{1 - \cos 2x}{x \sin 4x}$ approaches as $x \to 0$.

$1 - \cos 2x \approx \frac{1}{2}(2x)^2 = \frac{4}{2}x^2$

$\sin 4x \approx 4x$

Expression $\approx \frac{\frac{4}{2}x^2}{x \times 4x} = \frac{4}{8} = \frac{1}{2}$

Answer: $\frac{1}{2}$

7 Particle supported by two strings (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A 3 kg particle hangs in equilibrium from two light strings, as shown. Find the tension in each string (T_1 in the string at angle α). Take g = 9.8. Give answers to 3 s.f. if not exact.

$\sin \alpha = 0.6$, $\cos \alpha = 0.8$, $\sin \beta = \frac{12}{13}$, $\cos \beta = \frac{5}{13}$

Horizontal: $T_1 \cos \alpha = T_2 \cos \beta$, so $T_2 = 2.08\ T_1$

Vertical: $T_1 \sin \alpha + T_2 \sin \beta = 3\ g = 29.4$

$T_1(0.6 + 1.92) = 29.4$

$T_1 = 11.7$, $T_2 = 24.3$

Answer: $T_1 = 11.7$ N, $T_2 = 24.3$ N

8 Product moment correlation coefficient (AQA, Edexcel, OCR, MEI, Eduqas)

For n = 8 pairs of (x, y): $\Sigma x = 129$, $\Sigma y = 396$,
$\Sigma x^2 = 2605$, $\Sigma y^2 = 20358$, $\Sigma xy = 6945$.
Calculate the product moment correlation coefficient, and describe the correlation.

$S_{xx} = 2605 - \frac{129^2}{8} = 525$

$S_{yy} = 20358 - \frac{396^2}{8} = 756$

$S_{xy} = 6945 - \frac{129 \times 396}{8} = 560$

$r = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}} = 0.888$

This is strong positive correlation

Answer: r = 0.888, positive correlation

9 Recurrence relations: unknown first term (All boards)

A sequence is defined by $u_1 = k$ and $u_{n+1} = 2u_n - 1$.

Given that $u_3 = -7$, find (a) the value of k, (b) $\sum_{r=1}^{4} u_r$.

$u_2 = 2k - 1$, $u_3 = 2(2k - 1) - 1$

$4k - 3 = -7$ so $k = -1$

Terms: $-1, -3, -7, -15$

Sum $= -26$

Answer: (a) k = −1 (b) −26

10 Greatest and least values using R-form (All boards)

$g(x) = \frac{1}{30 + 24\sin x + 7\cos x}$

(a) Write $24\sin x + 7\cos x$ in the form $R\sin(x + \alpha)$, with α acute.

(b) Find the greatest and least values of $g(x)$ and the smallest positive x at which the greatest value occurs (1 d.p.).

$R = \sqrt{24^2 + 7^2} = 25$, $\alpha = \tan^{-1}\frac{7}{24} = 16.3°$

Denominator $= 30 + 25\sin(x + 16.3°)$, ranging from 5 to 55

Greatest $g = \frac{1}{5}$ (denominator least), least $g = \frac{1}{55}$

Least denominator when $\sin(x + 16.3°) = -1$: $x + 16.3° = 270°$

$x = 253.7°$

Answer: $R = 25$, $\alpha = 16.3°$; greatest $\frac{1}{5}$, least $\frac{1}{55}$; $x = 253.7°$