Day 362 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Polynomial division (All boards)

Find the quotient and the remainder when $6x^3 + x^2 - 27x - 16$ is divided by $(3x + 5)$.

Quotient (matching leading terms in turn): $2x^2 - 3x - 4$

$(3x + 5)(2x^2 - 3x - 4) = 6x^3 + x^2 - 27x - 20$

Remainder $= 6x^3 + x^2 - 27x - 16 - (6x^3 + x^2 - 27x - 20) = 4$

Answer: quotient $2x^2 - 3x - 4$, remainder 4

2 Differentiating sec, cosec, tan and cot (All boards)

$y = \cot(2x)$.

Find $\frac{dy}{dx}$ and its exact value at $x = \frac{\pi}{12}$.

$\frac{dy}{dx} = -2\operatorname{cosec}^2(2x)$

At $x = \frac{\pi}{12}$ the angle $2x = \frac{\pi}{6}$, so the gradient is -8

Answer: $\frac{dy}{dx} = -2\operatorname{cosec}^2(2x)$; -8

3 Venn diagrams and conditional probability (All boards)

$P(A) = 0.35$, $P(B) = 0.6$ and $P(A \cap B) = 0.05$.

Find

(a) $P(A \cup B)$

(b) $P(A' \cap B')$

(c) $P(B' \mid A')$

(a) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.35 + 0.6 - 0.05 = 0.9$

(b) $P(A' \cap B') = 1 - P(A \cup B) = 1 - 0.9 = 0.1$

(c) $P(B' \mid A') = \frac{P(A' \cap B')}{P(A')} = \frac{1 - 0.9}{1 - 0.35} = \frac{2}{13}$

Answer: (a) 0.9 (b) 0.1 (c) $\frac{2}{13}$

4 Lift and cable tension (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A lift of mass 400 kg carries a person of mass 60 kg. The cable tension is 4692 N and the lift is accelerating. Find (a) the magnitude and direction of the acceleration, (b) the force of the floor on the person. Take $g = 9.8$.

Whole system (mass 460 kg), taking upwards as positive: $T - 460g = 460a$

$4692 - 4508 = 460a$, so $a = 0.4$

(a) a is positive, so the acceleration is 0.4 m s^{-2} upwards

(b) Person: $R - 60g = 60a$, so $R = 60(9.8 + 0.4)$

Answer: (a) 0.4 m s^{-2} upwards (b) 612 N

5 Partial fractions: improper fraction (All boards)

Express $\frac{x^2 - 2x - 24}{(x - 2)(x + 2)}$ in the form $1 + \frac{A}{(x - 2)} + \frac{B}{(x + 2)}$, where A and B are constants to be found.

Divide: $x^2 - 2x - 24 = (x^2 - 4) + (-2x - 20)$

So the fraction is $1 + \frac{-2x - 20}{(x - 2)(x + 2)}$

$A(x + 2) + B(x - 2) \equiv -2x - 20$

$x = 2$ gives $A = -6$; $x = -2$ gives $B = 4$

Answer: $A = -6$, $B = 4$

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6 Inverse trigonometric functions (AQA, Edexcel, OCR, MEI, Eduqas)

$f(x) = 2 + 2\arccos(3x)$.

Find the exact value of x for which $f(x) = \frac{\pi + 6}{3}$.

Hence state the range of f.

$2\arccos(3x) = \frac{\pi}{3}$, so $\arccos(3x) = \frac{\pi}{6}$

$3x = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$, so $x = \frac{\sqrt{3}}{6}$

arccos takes values from 0 to π, so f(x) runs from 2 to $2\pi + 2$

Answer: $x = \frac{\sqrt{3}}{6}$; range $2 \leq f(x) \leq 2\pi + 2$

7 Normal then binomial (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The length of a plank is $N(75, 5^2)$. A plank is classed as 'special' if its length is less than 84. 12 planks are chosen at random. Find the probability that exactly 4 are special (3 s.f.).

$P(\text{special}) = P(X < 84) = P(Z < 1.80) = 0.964$

$Y \sim B(12, 0.964)$

$P(Y = 4) = C(12, 4)(0.964)^4(0.0359)^8 = 0.00000000119$

Answer: 0.00000000119

8 Equations with sec x and tan x (All boards)

Solve $3\sec^2 x + 9\tan x + 3 = 0$ for $0° \leq x \leq 360°$. Give answers to 1 d.p.

Use $\sec^2 x \equiv 1 + \tan^2 x$: $3\tan^2 x + 9\tan x + 6 = 0$

$\tan x = -2$ or $\tan x = -1$

$\tan x = -2$: $x = 116.6°, 296.6°$

$\tan x = -1$: $x = 135.0°, 315.0°$

Answer: $x = 116.6°, 135.0°, 296.6°, 315.0°$

9 Circle and line: tangent condition (AQA, Edexcel, OCR, MEI, Eduqas)

The line $12x + 5y = k$ is a tangent to the circle $(x + 2)^2 + (y + 2)^2 = 36$.

Find the two possible values of k.

Centre $(-2, -2)$, radius 6

Distance from centre to the line = radius:

$\frac{|-34 - k|}{13} = 6$

$|-34 - k| = 78$

Answer: $k = -112$ or $k = 44$

10 Catching up (delayed start) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Car A passes P at constant speed 12 m s^{-1}. 6 s later car B leaves P from rest, in the same direction, with acceleration 3 m s^{-2}. B overtakes A at Q. Find (a) the time B takes to reach Q, (b) PQ, (c) the speed of B at Q.

Let B take s seconds; A has then been moving for (s + 6) seconds.

B: $\frac{1}{2}\times 3\times s^2 = 1.5\,s^2$ A: $12(s + 6)$

$1.5\,s^2 - 12\,s - 72 = 0$ so $s = 12$ $(s > 0)$

(b) $PQ = 1.5\times 12^2 = 216$

(c) $v = 0 + at = 3\times 12 = 36$

Answer: (a) 12 s (b) 216 m (c) 36 m s^{-1}