Day 361 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Arithmetic sequences: finding a and d (All boards)

The 5th term of an arithmetic sequence is -29 and the 11th term is -65.
Find the first term and common difference, and the sum of the first 12 terms.

$u_{11} - u_5 = 6\,d = -65 - (-29)$ so $d = -6$

$a + 4\,d = -29$ so $a = -5$

$S_{12} = \frac{12}{2}(2 \times (-5) + 11 \times (-6))$

Answer: $a = -5$, $d = -6$, $S_{12} = -456$

2 Stratified sampling (AQA, Edexcel, OCR, MEI, Eduqas)

A stratified sample of 80 people is taken from three groups.
10 of the sample are from Sales, which has 200 people.
(a) Find the total number of people.
(b) Admin has 240 people. How many are sampled?

Sales sampled fraction $= \frac{10}{200} = \frac{1}{20}$

Total $= 80 \div \frac{1}{20} = 1600$

Admin: $240 \times \frac{1}{20} = 12$

Answer: (a) 1600 (b) 12

3 Equilibrium on a slope: range of force (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A 2 kg particle rests on a rough plane inclined at α, $\tan\alpha = \frac{8}{15}$, $\mu = 0.1$. A force of P N acts up a line of greatest slope and the particle stays at rest. Find the range of values of P. Take $g = 9.8$. Give answers to 3 s.f. if not exact.

$\sin\alpha = \frac{8}{17}$, $\cos\alpha = \frac{15}{17}$, $R = 2\,g\cos\alpha = \frac{294}{17}$ N

Limiting friction $= \mu R = \frac{147}{85}$ N

P smallest: friction acts up the slope: $P + \mu R = 2\,g\sin\alpha$, $P = \frac{784}{85} - \frac{147}{85}$

P greatest: friction acts down the slope: $P = \frac{784}{85} + \frac{147}{85}$

Answer: $7.49\text{ N} \leq P \leq 11.0\text{ N}$

4 Finding an unknown standard deviation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X is normally distributed with mean 39 and standard deviation σ.
Given that $P(X > 47) = 0.1587$, find σ to 3 significant figures.

$P(Z > z) = 0.1587$, so $z = 1.00$

$\frac{47 - 39}{\sigma} = 1.00$

$\sigma = \frac{8}{1.00} = 8.00$

Answer: $\sigma = 8.00$

5 Modulus graph and a line (All boards)

The diagram shows the graphs of $y = |x + 5|$ and $y = \frac{1}{2}x + 4$.

The graphs meet at A and B.
Find the coordinates of A and B.

$|x + 5| = \frac{1}{2}x + 4$

Solve $x + 5 = \pm(\frac{1}{2}x + 4)$.

$x = -6$ and $x = -2$ (both checked in the original equation)
$y = 1$ and $y = 3$
Answer: A$(-6, 1)$, B$(-2, 3)$

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6 Connected rates of change: cones and bowls (AQA, Edexcel, OCR, MEI, Eduqas)

Water pours into a cone, vertex down, at 40 cm^3s^{-1}. The surface radius is always $\frac{h}{2}$.

Find the rate at which the depth is rising when h = 5 cm.

$V = \frac{1}{3}\pi(h/2)^2h = \frac{\pi h^3}{12}$, so $\frac{dV}{dh} = \frac{\pi h^2}{4}$

$\frac{dh}{dt} = \frac{dV}{dt \div dV/dh} = \frac{40 \times 4}{\pi h^2}$

h = 5: $\frac{dh}{dt} = \frac{32}{5\pi}$ cm s^{-1}

Answer: $\frac{32}{5\pi}$ cm/s

7 Particles on two smooth inclined planes (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The system is released from rest. Find (a) the magnitude of the acceleration and which particle moves down, (b) the tension. Take g = 9.8. Give answers to 3 s.f. if not exact.

A: weight component along plane = 5 g sin 45°; B: 5 g sin 30°
A moves down, B up (5 sin 45° > 5 sin 30°): F = ma for the system
5 g sin 45° − 5 g sin 30° = (5 + 5)a
(a) a = 1.01
(b) For A: 5 g sin 45° − T = 5 a, so T = 29.6

Answer: (a) 1.01 m s^{-2}, A moves down (b) 29.6 N

8 Equations with cosec x and cot x (All boards)

Solve $3\text{cosec}^2 x - 9\cot x + 3 = 0$
for 0° < x < 360°. Give answers to 1 d.p.

Use $\text{cosec}^2 x \equiv 1 + \cot^2 x$: $3\cot^2 x - 9\cot x + 6 = 0$
cot x = 2 or cot x = 1

cot x = 2 means tan x = $\frac{1}{2}$: x = 26.6°, 206.6°

cot x = 1 means tan x = 1: x = 45.0°, 225.0°

Answer: x = 26.6°, 45.0°, 206.6°, 225.0°

9 Surds: equation with surd coefficients (AQA, Edexcel, OCR, MEI, Eduqas)

Solve $x(\sqrt{6} - 2) = 2 - 3\sqrt{6}$, writing x in the form $p + q\sqrt{6}$.

$x = \frac{2 - 3\sqrt{6}}{\sqrt{6} - 2}$

Multiply top and bottom by $\sqrt{6} + 2$.

Numerator: $(2 - 3\sqrt{6})(\sqrt{6} + 2) = -14 - 4\sqrt{6}$

Denominator: 6 − 4 = 2

Answer: $x = -7 - 2\sqrt{6}$

10 Integration by substitution (All boards)

Use the substitution $u = x^2 + 1$ to find the exact value of

$\int_0^1 x(x^2 + 1)^3 \, dx$.

$du = 2x\,dx$, so $x\,dx = \frac{1}{2}du$; x = 0 gives u = 1, x = 1 gives u = 2

$I = \frac{1}{2}\int_1^2 u^3 \, du = \frac{1}{2}[\frac{u^4}{4}]$

$= \frac{15}{8}$

Answer: $\frac{15}{8}$