Day 360 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Change of sign to locate a root (All boards)

$f(x) = x^2 - 4\cos x - 7$.
Show that the equation $f(x) = 0$ has a root α in the interval $[2, 3]$ (x in radians).

$f(x)$ is continuous on the interval
$f(2) = -1.34 < 0$
$f(3) = 5.96 > 0$
A change of sign, so a root lies in $[2, 3]$
Answer: Sign change: $f(2) = -1.34$, $f(3) = 5.96$

2 Arithmetic sequences: finding a and d (All boards)

The 2nd term of an arithmetic sequence is 11 and the 9th term is 60.
Find the first term and common difference, and the sum of the first 15 terms.

$u_9 - u_2 = 7\,d = 60 - 11$ so $d = 7$
$a + 1\,d = 11$ so $a = 4$
$S_{15} = \frac{15}{2}(2 \times 4 + 14 \times 7)$
Answer: $a = 4$, $d = 7$, $S_{15} = 795$

3 Exact definite integrals of standard functions (All boards)

Find the exact value of

$\int_0^2 (2x + 1)^2\,dx$.

Integrate: $[\frac{(2x + 1)^3}{6}]$

Substitute the limits and subtract: $\frac{62}{3}$

Answer: $\frac{62}{3}$

4 Poisson probabilities (CIE)

Emails arrive at a mean rate of 2.5 per day, as a Poisson process.
Find the probability that in one such period there are
(a) exactly 3 (b) at least 3 (c) fewer than 3 (3 s.f.).

$X \sim Po(2.5)$
(a) $P(X = 3) = e^{-2.5}\,2.5^3/6! = 0.214$
(b) $1 - P(X \leq 2) = 1 - 0.544 = 0.456$
(c) $P(X \leq 2) = 0.544$
Answer: (a) 0.214 (b) 0.456 (c) 0.544

5 Addition formulae with given sin and cos (All boards)

A is acute and B is obtuse, with $\sin A = \frac{4}{5}$ and $\cos B = -\frac{12}{13}$.
Find the exact values of $\sin(A+B)$ and $\cos(A-B)$.

$\cos A = \frac{3}{5}$ (A acute), $\sin B = \frac{5}{13}$ (B obtuse), so $\tan A = \frac{4}{3}$, $\tan B = -\frac{5}{12}$

$\sin(A+B) = \sin A \cos B + \cos A \sin B = \frac{4}{5} \times (-\frac{12}{13}) + \frac{3}{5} \times \frac{5}{13} = -\frac{33}{65}$

$\cos(A-B) = \cos A \cos B + \sin A \sin B = \frac{3}{5} \times (-\frac{12}{13}) + \frac{4}{5} \times \frac{5}{13} = -\frac{16}{65}$

Answer: $\sin(A+B) = -\frac{33}{65}$, $\cos(A-B) = -\frac{16}{65}$

6 Foot of a perpendicular (All boards)

The line l has equation $x - 2y = -1$.
(a) Find the coordinates of the foot of the perpendicular from P(1, 6) to l.
(b) Hence find the shortest distance from P to l, in exact form.

Gradient of l = $\frac{1}{2}$, so perpendicular gradient = -2
Perpendicular through P: $-2x - y = -8$
Solve with l: $x = 3$, $y = 2$
Distance^2 = $(1 - 3)^2 + (6 - 2)^2 = 20$
Answer: (a) (3, 2) (b) $2\sqrt{5}$

7 Friction: find μ then stopping distance (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A 10 kg box is pulled from rest along rough horizontal ground by a horizontal force of 30.5 N and moves 7.5 m in 5 s. Find (a) the coefficient of friction. The force is then removed; (b) find the further distance moved. Take g = 9.8. Give answers to 3 s.f. if not exact.

$s = \frac{1}{2}at^2$: $7.5 = \frac{1}{2}a \times 5^2$, so $a = 0.6$
(a) $P - \mu 10\,g = 10\,a$: $30.5 - 98\,\mu = 6$, so $\mu = 0.25$
(b) $v = at = 0.6 \times 5 = 3$; after removal deceleration = $\mu g = 2.45$
$0 = 3^2 - 2 \times 2.45 \times s$, $s = 1.84$
Answer: (a) $\mu = 0.25$ (b) 1.84 m

8 Normal approximation to the binomial (All boards)

X ~ B(200, 0.25). Use a normal approximation to estimate P(X ≤ 50) to 3 significant figures.

np = 50, npq = 37.5, so Y ~ N(50, 37.5)
With continuity correction: P(Y < 50.5)
$z = \frac{50.5 - 50}{6.12} = 0.0816$, P(Z < 0.0816) = 0.533
Answer: 0.533

9 Two balls meeting (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Ball A is dropped from rest 34.3 m above the ground. At the same instant ball B is projected upwards at 34.3 m s^{-1} from the ground directly below A. Find (a) the time when they collide, (b) the height of the collision. Take g = 9.8.

Let the time be t. Distance fallen by A: $\frac{1}{2}gt^2 = 4.9t^2$
Height of B above the ground: $34.3\,t - 4.9t^2$
They meet when the distances add up to 34.3: $4.9t^2 + (34.3\,t - 4.9t^2) = 34.3$
so $34.3\,t = 34.3$, $t = 1$
Height of B = $34.3 \times 1 - 4.9 \times 1^2 = 29.4$
Answer: (a) 1 s (b) 29.4 m

10 Circle and line: tangent condition (AQA, Edexcel, OCR, MEI, Eduqas)

The line $-3x + 4y = k$ is a tangent to the circle
$(x - 1)^2 + (y - 2)^2 = 25$.
Find the two possible values of k.

Centre (1, 2), radius 5
Distance from centre to the line = radius:
$\frac{|5 - k|}{5} = 5$
$|5 - k| = 25$
Answer: $k = -20$ or $k = 30$