Day 36 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Arithmetic sequences: finding a and d (All boards)

The 6th term of an arithmetic sequence is 37 and the 12th term is 67.
Find the first term and common difference, and the sum of the first 20 terms.

$u_{12} - u_6 = 6\,d = 67 - 37$ so $d = 5$

$a + 5\,d = 37$ so $a = 12$

$S_{20} = \frac{20}{2}(2 \times 12 + 19 \times 5)$

Answer: a = 12, d = 5, S_{20} = 1190

2 Product rule: stationary points (All boards)

$y = (x - 5)e^{-x}$.
Find the exact coordinates of the stationary point and determine its nature.

$\frac{dy}{dx} = 1\,e^{-x} - (x - 5)e^{-x} = (-x + 6)e^{-x}$

$e^{-x} > 0$, so $\frac{dy}{dx} = 0$ gives $-x + 6 = 0$ and $x = 6$

$y = e^{-6}$; $\frac{d^2y}{dx^2}$ is negative there, so it is a maximum

Answer: (6, e^{-6}), maximum

3 Bayes' theorem in context (All boards)

5% of people have a fault. The inspection test is positive for 90% of those who have it and for 1% of those who do not.
(a) Find the probability that a person tests positive.
(b) Given a positive test, find the probability that the person has the fault (3 s.f.).

(a) $P(+) = 0.05 \times 0.9 + 0.95 \times 0.01$
$= 0.045 + 0.0095 = 0.0545$

(b) $P(\text{has it} \mid +) = \frac{0.05 \times 0.9}{0.0545}$

$= 0.826$

Answer: (a) 0.0545 (b) 0.826

4 Stopping distance (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A driver at constant speed sees a hazard 30 m ahead. After a reaction time of 1 s she brakes with deceleration 4 m s^{-2} and stops exactly at the hazard. Find (a) her initial speed, (b) the total time from seeing the hazard to stopping.

Let the initial speed be v. Reaction distance = 1 v, braking distance = $v^2 \div (2 \times 4)$
$1\,v + v^2 \div 8 = 30$, so $v^2 + 8\,v - 240 = 0$
$(v - 12)(v + 20) = 0$, so $v = 12$
(b) Braking time = $12 \div 4 = 3$ s; total = 1 + 3

Answer: (a) 12 m s^{-1} (b) 4 s

5 Arithmetic series: finding n (All boards)

An arithmetic series has first term 14 and common difference 6.
The sum of the first n terms is 840. Find n.

$\frac{n}{2}(2 \times 14 + (n - 1) \times 6) = 840$

$6n^2 + 22n = 1680$
$6n^2 + 22n - 1680 = 0$

$(n - 15)(6n + 112) = 0$, so $n = 15$ or $n = -\frac{56}{3}$

n must be a positive integer, so n = 15

Answer: 15

6 Binomial mean and variance (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X has a binomial distribution with mean 9 and variance 3.6.
(a) Find n and p.
(b) Find P(X = 7) to 3 significant figures.

(a) np = 9 and npq = 3.6

$q = \frac{npq}{np} = \frac{3.6}{9} = 0.4$, so p = 0.6

$n = \frac{9}{0.6} = 15$

(b) $P(X = 7) = C(15, 7)(0.6)^7(0.4)^8 = 0.118$

Answer: (a) n = 15, p = 0.6 (b) 0.118

7 Geometric series: smallest n (All boards)

A salary is £25000 in year 1 and increases by 4% each year.
In which year does the salary first exceed £37500?

Salary in year $n = 25000 \times 1.04^{n-1}$

$1.04^{n-1} > \frac{3}{2}$

$n - 1 > \frac{\log \frac{3}{2}}{\log 1.04} = 10.34$

n = 12

Answer: year 12

8 Connected rates of change: cones and bowls (AQA, Edexcel, OCR, MEI, Eduqas)

A hemispherical bowl of radius 10 cm is filled at 50 cm^3s^{-1}. When the depth is h, $V = \pi(10h^2 - \frac{1}{3}h^3)$.

Find $\frac{dh}{dt}$ when h = 4 cm.

$\frac{dV}{dh} = \pi(20h - h^2) = \pi h(20 - h)$

$\frac{dh}{dt} = \frac{50}{\pi h(20 - h)}$

h = 4: $\frac{dh}{dt} = \frac{50}{\pi \times 4 \times 16} = \frac{25}{32\pi}$ cm s^{-1}

Answer: $\frac{25}{32\pi}$ cm/s

9 Optimisation problems (All boards)

An open-topped box has a square base of side x cm and volume 32 cm^3.

Show that its surface area is $A = x^2 + \frac{128}{x}$, and find the least possible value of A, justifying that it is a minimum.

$h = \frac{32}{x^2}$, so $A = x^2 + 4xh = x^2 + \frac{128}{x}$

$\frac{dA}{dx} = 2x - \frac{128}{x^2} = 0$, so $x^3 = 64$ and x = 4

$\frac{d^2A}{dx^2} = 2 + \frac{256}{x^3} > 0$, so a minimum

$A = 16 + \frac{128}{4} = 48$

Answer: A_min = 48 cm^2 (x = 4)

10 Pulley, string goes slack (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Particles A (5 kg) and B (3 kg) hang on a light string over a smooth pulley, A being 1.5 m above the ground. Released from rest; A lands without rebounding. Find (a) the tension while both move, (b) the greatest height of B above its start. Take g = 9.8.

A: $5g - T = 5a$; B: $T - 3g = 3a$
$a = 2g \div 8 = 2.45$, so $T = 3(g + a) = 36.75$
When A lands, B has risen 1.5 m with $v^2 = 2 \times 2.45 \times 1.5 = 7.35$
B then moves freely under gravity: extra height $= v^2 \div (2 \times 9.8) = 0.375$
Total = 1.5 + 0.375
Answer: (a) 36.75 N (b) 1.875 m