Day 359 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Index equations solved with logarithms (All boards)

Solve $2^{3x-2} = 7^{3x+3}$.
Give your answer to 3 significant figures.

Take logs: $(3x - 2)\ln 2 = (3x + 3)\ln 7$
Collect x terms: $x(3\ln 2 - 3\ln 7) = 3\ln 7 + 2\ln 2$
$x = \dfrac{3\ln 7 + 2\ln 2}{3\ln 2 - 3\ln 7}$
$x = -1.92$
Answer: x = −1.92

2 Comparing data sets from the large data set (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Summary statistics for the daily total rainfall (mm) in the same month are:
Hurn: mean 21.5, standard deviation 5.3;
Beijing: mean 8, standard deviation 5.
Make two comparisons.

Means: Hurn has the higher mean (21.5 against 8), so on average the total rainfall is greater at Hurn
Standard deviations: Hurn is larger at Hurn (5.3 against 5)
so the readings at Hurn are more variable and those at Beijing are more consistent
Answer: Hurn higher on average; Hurn more variable

3 Circle: centre, radius and position of a point (AQA, Edexcel, OCR, MEI, Eduqas)

A circle has equation $x^2 + y^2 + 12x - 10y + 51 = 0$.
(a) Find the centre and radius of the circle.
(b) Determine whether P(−6, −1) lies inside or outside the circle.

Complete the square: $(x + 6)^2 + (y - 5)^2 = 10$
Centre (−6, 5), radius $\sqrt{10}$
Distance^2 from centre to P = $0^2 + (-6)^2 = 36$
Compare with $r^2 = 10$
Answer: (a) centre (−6, 5), radius $\sqrt{10}$ (b) P is outside the circle

4 Non-uniform rod and equal tensions (AQA, Edexcel, OCR, MEI)

A non-uniform rod AB (length 5 m, mass 10 kg) has its centre of mass at G, AG = 1.75 m. It hangs horizontally from vertical strings at A and B. A particle of mass m kg is fixed at P, AP = 3 m, making the tensions equal. Find (a) m, (b) the tension. Take g = 9.8.

Equal tensions T: $2T = (10 + m)g$, so $T = \frac{1}{2}(10 + m)g$
Moments about A: $T\times 5 = 10\,g\times 1.75 + m\,g\times 3$
$\frac{1}{2}(10 + m)\times 5 = 17.5 + 3m \rightarrow 25 + 2.5m = 17.5 + 3m$
$m(3 - 2.5) = 25 - 17.5$, so $m = 15$
(b) $T = \frac{1}{2}(10 + 15)\times 9.8$
Answer: (a) m = 15 (b) 122.5 N

5 Logarithmic equations (All boards)

Solve $\log_2(x + 5) + \log_2(x - 7) = 6$.

Combine: $\log_2[(x + 5)(x - 7)] = 6$, so $(x + 5)(x - 7) = 2^6 = 64$
$x^2 - 2x - 99 = 0$: $(x - 11)(x + 9) = 0$
$x = 11$ or $x = -9$; $x = -9$ makes $x - 7$ negative, so it is rejected
Answer: x = 11

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6 Circle through two points with centre on a line (AQA, Edexcel, OCR, MEI, Eduqas)

A circle passes through A(5, −7) and B(−3, −7).
Its centre lies on the line $y = 3x - 7$.
Find the equation of the circle.

Centre $(h, 3h - 7)$ is equidistant from A and B.
$(h - 5)^2 + (3h)^2 = (h + 3)^2 + (3h)^2$
Solving gives $h = 1$, so centre $(1, -4)$
$r^2 = (4)^2 + (-3)^2 = 25$
Answer: $(x - 1)^2 + (y + 4)^2 = 25$

7 Maximum speed from velocity function (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves in a straight line with velocity $v = (-2t^2 + 12t)$ m s^{-1} at time t s, $t \geq 0$, starting at O. Find (a) its greatest speed, (b) its distance from O when $t = 6$, (c) the time at which it returns to O.

(a) $dv/dt = 12 - 4t = 0$, so $t = 3$ (a maximum since the t^2 coefficient is negative)
$v = 12 \times 3 - 2 \times 3^2 = 18$
(b) $x = \int v\,dt = 6t^2 - \frac{2}{3}t^3$
$x(6) = 72$
(c) $x = 0$ when $t^2(6 - \frac{2}{3}t) = 0$, so $t = 9$
Answer: (a) 18 m s^{-1} (b) 72 m (c) 9 s

8 Differential equations in context (All boards)

A drink at 75°C is left in a room at 15°C. Its temperature θ °C satisfies $\frac{d\theta}{dt} = -k(\theta - 15)$.
After 5 minutes the temperature is 60°C. Find k, then the time for it to reach 30°C, to 3 s.f.

$\int \frac{1}{\theta - 15}\,d\theta = \int -k\,dt$, so $\ln(\theta - 15) = -kt + C$
$t = 0$: $\theta - 15 = 60$, so $\theta - 15 = 60e^{-kt}$
$t = 5$: $45 = 60e^{-5k}$, so $k = \frac{\ln(\frac{4}{3})}{5} = 0.0575$
$15 = 60e^{-kt}$: $t = \frac{\ln(4)}{k} = 24.1$ minutes
Answer: k = 0.0575, t = 24.1 minutes

9 Inverse of a quadratic function (All boards)

$f(x) = 2x^2 - 16x + 34$, $x \geq 4$.
Find $f^{-1}(x)$ and state its domain.

$f(x) = 2(x - 4)^2 + 2$
Let $y = f(x)$: $(x - 4)^2 = \frac{y - 2}{2}$
$x \geq 4$, so take the positive root: $x = 4 + \sqrt{\frac{y - 2}{2}}$
Answer: $f^{-1}(x) = 4 + \sqrt{\frac{x - 2}{2}}$, $x \geq 2$

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10 Expectation and variance of linear combinations (Edexcel, OCR, MEI, Eduqas, CIE)

X has E(X) = 4 and Var(X) = 3.

(a) Find $E(X^2)$.

(b) $Y = 5X + 3$. Find E(Y) and Var(Y).

(c) X_1 and X_2 are independent copies of X. Find $Var(X_1 - X_2)$.

(a) $E(X^2) = Var(X) + [E(X)]^2 = 3 + 16 = 19$

(b) $E(Y) = 5 \times 4 + 3 = 23$, $Var(Y) = (5)^2 \times 3 = 75$

(c) $Var(X_1 - X_2) = Var(X_1) + Var(X_2) = 2 \times 3 = 6$

Answer: (a) 19 (b) E(Y) = 23, Var(Y) = 75 (c) 6