**Day 358** **A Level Maths: A Tier 10-a-day: Model Solutions**

## 1 Binomial expansion: a coefficient (All boards)

Find the coefficient of $x^2$ in the expansion of $(4 + 3x)^8$.

Term in $x^2$: ${}^8C_2 \times 4^6 \times (3x)^2$

$= 28 \times 4096 \times 9$

**Answer: 1032192**

## 2 Comparing data sets from the large data set (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Summary statistics for the daily total rainfall (mm) in the same month are:
Hurn: mean 20.5, standard deviation 1.5;
Beijing: mean 29.5, standard deviation 3.5.
Make two comparisons.

Means: Beijing has the higher mean (29.5 against 20.5), so on average the total rainfall is greater at Beijing
Standard deviations: Beijing is larger at Beijing (3.5 against 1.5)
so the readings at Beijing are more variable and those at Hurn are more consistent

**Answer: Beijing higher on average; Beijing more variable**

## 3 Speed-time graphs (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the speed-time graph for a train journey of 1400 m. Find (a) V, (b) the deceleration in the final stage.

(a) Distance = area of trapezium = $\frac{1}{2}(90 + 50) \times V$

$1400 = 70\,V$, so $V = 20$

(b) Deceleration = $V \div 20 = 20 \div 20$

**Answer: (a) V = 20 (b) 1 m s$^{-2}$**

## 4 Discrete random variable with unknown constant (Edexcel, OCR, MEI, Eduqas, CIE)

The random variable X has $P(X = x) = kx^2$ for $x = 2, 3, 4, 5, 6$.

(a) Show that $k = \frac{1}{90}$. (b) Find E(X) and Var(X).

(c) Find $P(X \geq 4)$.

(a) $\Sigma\, P(X = x) = 1$: $4 + 9 + 16 + 25 + 36 = 90$, so $90k = 1$

(b) $E(X) = 2 \times \frac{2}{45} + 3 \times \frac{1}{10} + 4 \times \frac{8}{45} + 5 \times \frac{5}{18} + 6 \times \frac{2}{5} = \frac{44}{9}$

$E(X^2) = \frac{379}{15}$, so $Var(X) = \frac{379}{15} - (\frac{44}{9})^2 = \frac{553}{405}$

(c) $P(X \geq 4) = \frac{8}{45} + \frac{5}{18} + \frac{2}{5} = \frac{77}{90}$

**Answer: (b) E(X) = $\frac{44}{9}$, Var(X) = $\frac{553}{405}$ (c) $\frac{77}{90}$**

## 5 Equations with sin 2x and a trig function (All boards)

Solve $\sin 2x = \sqrt{2} \sin x$ for $-180° \leq x \leq 180°$.

$\sin 2x \equiv 2\sin x \cos x$: $\sin x\,(2\cos x - \sqrt{2}) = 0$

$\sin x = 0$: $x = -180°, 0°, 180°$

$\cos x = \frac{\sqrt{2}}{2}$: $x = -45°, 45°$

Do not divide by sin x, as that would lose solutions

**Answer: x = −180°, −45°, 0°, 45°, 180°**

## 6 Circle touching both axes (AQA, Edexcel, OCR, MEI, Eduqas)

A circle touches both coordinate axes and passes through the point (5, −10). Its centre lies in the fourth quadrant.
Find the equations of the two possible circles.

Centre (r, −r), radius $r > 0$.

Through (5, −10): $(5 - r)^2 + (-10 + r)^2 = r^2$

$r^2 - 30r + 125 = 0$

$(r - 5)(r - 25) = 0$, so r = 5 or 25

**Answer: $(x - 5)^2 + (y + 5)^2 = 25$ or $(x - 25)^2 + (y + 25)^2 = 625$**

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## 7 Equations with cosec x and cot x (All boards)

Solve $4\text{cosec}^2 x - 4\cot x - 12 = 0$
for $0° < x < 180°$. Give answers to 1 d.p.

Use $\text{cosec}^2 x \equiv 1 + \cot^2 x$: $4\cot^2 x - 4\cot x - 8 = 0$
$\cot x = -1$ or $\cot x = 2$
$\cot x = -1$ means $\tan x = -1$: $x = 135.0°$
$\cot x = 2$ means $\tan x = \frac{1}{2}$: $x = 26.6°$

**Answer: x = 26.6°, 135.0°**

## 8 Sigma notation (All boards)

Given that $\sum_{r=1}^{n}(4r + 2) = 240$, find the value of n.

$$\sum_{r=1}^{n}(4r + 2) = 4 \times \frac{n(n + 1)}{2} + 2n$$

$2n^2 + 4n = 240$
$4n^2 + 8n - 480 = 0$
Solve and reject the negative root: $n = 10$
**Answer: 10**

## 9 Projectile through a given point (AQA, Edexcel, OCR, MEI, Eduqas)

A ball is projected from a point O on horizontal ground at angle $\alpha$ above the horizontal, $\tan\alpha = \frac{4}{3}$, and passes through A, 36 m horizontally from O and 28.4 m above the ground. Find the speed of projection. Take $g = 9.8$.

Let the speed be U. Horizontal: $36 = U\cos\alpha \times t$, so $t = 36 \div (0.6\,U)$
Vertical: $28.4 = U\sin\alpha \times t - 4.9t^2$
Eliminate t: $y = x\tan\alpha - 4.9x^2 \div (U^2\cos^2\alpha)$
$28.4 = 48 - 4.9\times36^2 \div (0.36\,U^2)$
$U^2 = 17640 \div 19.6 = 900$, $U = 30$
**Answer: 30 m s$^{-1}$**

## 10 Foot of perpendicular and distance to a line (CIE (9709); Further Maths on other boards)

The line l has equation $\mathbf{r} = \begin{pmatrix}0\\-6\\1\end{pmatrix} + t\begin{pmatrix}1\\2\\1\end{pmatrix}$.

Find the coordinates of the foot of the perpendicular from P(4, −2, 1) to l, and the exact distance from P to l.

Foot F = A + t d: $\mathbf{PF} = \begin{pmatrix}t-4\\2t-4\\t\end{pmatrix}$
$\mathbf{PF}\cdot\mathbf{d} = 0$ gives $6t - 12 = 0$, so $t = 2$
F = (2, −2, 3); $\mathbf{PF} = \begin{pmatrix}-2\\0\\2\end{pmatrix}$, so $|\mathbf{PF}| = \sqrt{8} = 2\sqrt{2}$

**Answer: F(2, −2, 3), distance $2\sqrt{2}$**