**Day 356** **A Level Maths: A Tier 10-a-day: Model Solutions**

## 1 Partial fractions: distinct linear factors (All boards)

Express $\frac{3x + 10}{(2x + 3)(x - 4)}$ in partial fractions.

Let $\frac{3x + 10}{(2x + 3)(x - 4)} \equiv \frac{A}{(2x + 3)} + \frac{B}{(x - 4)}$

$3x + 10 \equiv A(x - 4) + B(2x + 3)$

Substitute $x = -\frac{3}{2}$: $A = -1$

Substitute $x = 4$: $B = 2$

**Answer:** $\mathbf{\frac{-1}{(2x + 3)} + \frac{2}{(x - 4)}}$

## 2 Constant acceleration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A cyclist passes P with speed 3 m $s^{-1}$ and accelerates uniformly. She travels 90 m in the next 15 s, reaching Q. Find (a) her acceleration, (b) her speed at Q, (c) her speed at the midpoint of PQ. Give answers to 3 s.f. if not exact.

(a) $s = ut + \frac{1}{2}at^2$: $90 = 3\times15 + \frac{1}{2}\times a\times15^2$, so $a = 0.4$

(b) $v = u + at = 3 + 0.4\times15 = 9$

(c) For the midpoint, $s = 45$: $v^2 = u^2 + 2as = 3^2 + 2\times0.4\times45$

$v^2 = 45$

**Answer: (a) 0.4 m $s^{-2}$ (b) 9 m $s^{-1}$ (c) 6.71 m $s^{-1}$**

## 3 Double and triple angle values (All boards)

$\theta$ is obtuse and $\sin\theta = \frac{21}{29}$.

Find the exact values of $\sin 2\theta$, $\cos 2\theta$ and $\tan 2\theta$.

$\cos\theta = -\sqrt{1 - \left(\frac{21}{29}\right)^2} = -\frac{20}{29}$ (obtuse)

$\sin 2\theta = 2\sin\theta\cos\theta = 2 \times \frac{21}{29} \times \left(-\frac{20}{29}\right) = -\frac{840}{841}$

$\cos 2\theta = 1 - 2\sin^2\theta = 1 - 2 \times \left(\frac{21}{29}\right)^2 = -\frac{41}{841}$

$\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta} = \frac{840}{41}$

**Answer:** $\mathbf{\sin 2\theta = -\frac{840}{841}, \cos 2\theta = -\frac{41}{841}, \tan 2\theta = \frac{840}{41}}$

## 4 Completing the square (All boards)

$f(x) = -x^2 - 4x + 1$.

(a) Write f(x) in the form $a(x + p)^2 + q$.

(b) State the maximum value of f(x) and the value of x at which it occurs.

(c) Solve $f(x) = 0$, giving surds in simplest form.

$f(x) = -(x + 2)^2 + 5$

(b) maximum value 5 when $x = -2$

(c) $-(x + 2)^2 = -5$, so $(x + 2)^2 = 5$

$x + 2 = \pm\sqrt{5}$

**Answer: (a)** $\mathbf{-(x + 2)^2 + 5}$ **(b) maximum 5 at** $\mathbf{x = -2}$ **(c)** $\mathbf{x = -2 \pm \sqrt{5}}$

## 5 Conditional probability without replacement (All boards)

A bag has 6 black and 2 white counters. Two are taken at random without replacement.
Given that the counters are different colours, find the probability that the first is black.

P(black then white) = $\frac{6}{8} \times \frac{2}{7} = \frac{3}{14}$

P(white then black) = $\frac{2}{8} \times \frac{6}{7} = \frac{3}{14}$

P(different) = $\frac{3}{7}$

P(first black | different) = $\frac{\frac{3}{14}}{\frac{3}{7}}$

= $\frac{1}{2}$

**Answer:** $\frac{1}{2}$

## 6 Circle: centre, radius and position of a point (AQA, Edexcel, OCR, MEI, Eduqas)

A circle has equation $x^2 + y^2 + 6x + 4y + 4 = 0$.
(a) Find the centre and radius of the circle.
(b) Determine whether P(1, −4) lies inside or outside the circle.

Complete the square: $(x + 3)^2 + (y + 2)^2 = 9$
Centre (−3, −2), radius 3
Distance$^2$ from centre to P = $4^2 + (-2)^2 = 20$
Compare with $r^2 = 9$
**Answer: (a) centre (−3, −2), radius 3 (b) P is outside the circle**

## 7 Newton-Raphson method (AQA, Edexcel, OCR, MEI, Eduqas)

$f(x) = x^2 - 6$. Show that the Newton-Raphson formula gives $x_{n+1} = \frac{1}{2}(x_n + \frac{6}{x_n})$,

and use it with $x_0 = \frac{5}{2}$ to find $x_2$ as an approximation to $\sqrt{6}$, to 4 d.p.

$f'(x) = 2x$, so $x_{n+1} = x_n - \frac{x_n^2 - 6}{2x_n} = \frac{1}{2}(x_n + \frac{6}{x_n})$

$x_1 = \frac{1}{2}(\frac{5}{2} + \frac{6}{\frac{5}{2}}) = \frac{49}{20}$

$x_2 = 2.4495$
**Answer: $x_2 = 2.4495$**

## 8 Beam about to tilt (AQA, Edexcel, OCR, MEI)

A uniform plank AB (mass 10 kg, length 4 m) rests horizontally on supports at C and D, where AC = 1 m and DB = 0.5 m.
A 10 kg particle is placed at E, AE = x m, and the plank is about to tilt about C. Find x.

About to tilt about C, so the reaction at D is zero and E lies between A and C.
Moments about C: 10 g×(1 − x) = 10 g×(2 − 1)
10(1 − x) = 10
x = 1 − 1
**Answer: x = 0**

## 9 Stationary points with trigonometric and exponential terms (All boards)

$y = e^{2x} - 6e^x + 4x$.

Show that the stationary points occur where $e^x = 1$ and $e^x = 2$, and find the exact y-coordinate of each, with its nature.

$\frac{dy}{dx} = 2e^{2x} - 6e^x + 4 = 2(e^x - 1)(e^x - 2)$

So $e^x = 1$ or $e^x = 2$, i.e. $x = \ln 1$ or $x = \ln 2$

$\frac{d^2y}{dx^2} = 4e^{2x} - 6e^x = e^x(4e^x - 6)$ changes sign: negative at the smaller root, positive at the larger

**Answer: $x = 0$, $y = -5$ (maximum); $x = \ln 2$, $y = 4 \ln 2 - 8$ (minimum)**

## 10 Binomial in two stages (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Bulbs are defective with probability 0.25, independently. A pack of 7 bulbs is rejected if it has at least 3 defective. From 6 such packs, find the probability that exactly 2 are rejected (3 s.f.).

$Y \sim B(7, 0.25)$. $P(\text{rejected}) = P(Y \geq 3) = 1 - 0.756 = 0.244$

$W \sim B(6, 0.244)$ is the number rejected

$P(W = 2) = C(6, 2)(0.244)^2(1 - 0.244)^4 = 0.291$

**Answer: 0.291**