Day 355 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Projectile through a given point (AQA, Edexcel, OCR, MEI, Eduqas)

A ball is projected from a point O on horizontal ground at angle α above the horizontal, $\tan\alpha = \frac{12}{5}$, and passes through A, 20 m horizontally from O and 31.1 m above the ground. Find the speed of projection. Take g = 9.8.

Let the speed be U. Horizontal: $20 = U\cos\alpha \times t$, so $t = 20 \div (\frac{5}{13}\ U)$

Vertical: $31.1 = U\sin\alpha \times t - 4.9t^2$

Eliminate t: $y = x\tan\alpha - 4.9x^2 \div (U^2\cos^2\alpha)$

$31.1 = 48 - 4.9\times20^2 \div (\frac{25}{169}\ U^2)$

$U^2 = 13249.6 \div 16.9 = 784$, $U = 28$

Answer: 28 m s^{-1}

2 Comparing data sets from the large data set (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Summary statistics for the daily mean windspeed (knots) in the same month are:
Heathrow: mean 15.5, standard deviation 1.3;
Leuchars: mean 15, standard deviation 3.6.
Make two comparisons.

Means: Heathrow has the higher mean (15.5 against 15), so on average the mean windspeed is greater at Heathrow

Standard deviations: Leuchars is larger at Leuchars (3.6 against 1.3)

so the readings at Leuchars are more variable and those at Heathrow are more consistent

Answer: Heathrow higher on average; Leuchars more variable

3 Passing a point twice (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A stone is projected vertically upwards from the ground. It passes a ledge 2 s after projection and again 5 s after projection. Find (a) the speed of projection, (b) the height of the ledge, (c) the greatest height. Take g = 9.8.

Same height at t = 2 and t = 5, so the stone is at its highest point at t = 3.5

(a) At the top v = 0: $0 = u - 9.8\times3.5$, so u = 34.3

(b) $s = ut - 4.9t^2$ with t = 2: $s = 34.3\times2 - 4.9\times2^2 = 49$

(c) $v^2 = u^2 - 2gs$: $0 = 34.3^2 - 19.6s$, s = 60.025

Answer: (a) 34.3 m s^{-1} (b) 49 m (c) 60.025 m

4 Partial fractions: distinct linear factors (All boards)

Express $\frac{-15x - 30}{(2x + 3)(x + 3)}$ in partial fractions.

Let $\frac{-15x - 30}{(2x + 3)(x + 3)} \equiv \frac{A}{(2x + 3)} + \frac{B}{(x + 3)}$

$-15x - 30 \equiv A(x + 3) + B(2x + 3)$

Substitute $x = -\frac{3}{2}$: A = −5

Substitute x = −3: B = −5

Answer: $\frac{-5}{(2x + 3)} + \frac{-5}{(x + 3)}$

5 Integration by substitution (All boards)

Use the substitution $u = x^2$ to find the exact value of

$\int_0^2 x\,e^{2x^2}\,dx$.

$du = 2x\,dx$, so $x\,dx = \frac{1}{2}du$; limits: u = 0 to u = 4

$I = \frac{1}{2}\int_0^4 e^{2u}\,du = \frac{1}{2}[\frac{e^{2u}}{2}]$

$= \frac{e^8 - 1}{4}$

Answer: $\frac{e^8 - 1}{4}$

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6 Binomial expansion: product with a bracket (All boards)

Find the coefficient of x^3 in the expansion of
$(2 + 3x)(2 - 2x)^8$.

In $(2 - 2x)^8$: coefficient of x^2 is 7168, of x^3 is -14336
Coefficient = $2 \times (-14336) + 3 \times 7168$
Answer: -7168

7 Integration by parts (All boards)

Use integration by parts to find the exact value of

$\int_1^e x \ln x \, dx$.

$u = \ln x$, $\frac{dv}{dx} = x$: $\int x \ln x \, dx = [\frac{x^2 \ln x}{2}] - \int \frac{x^1}{2} \, dx$

$= [\frac{x^2 \ln x}{2} - \frac{x^2}{4}]$

$= \frac{e^2 + 1}{4}$

Answer: $\frac{e^2 + 1}{4}$

8 Bayes' theorem in context (All boards)

1% of people have a disease. The screening test is positive for 98% of those who have it and for 1% of those who do not.
(a) Find the probability that a person tests positive.
(b) Given a positive test, find the probability that the person has the disease (3 s.f.).

(a) $P(+) = 0.01 \times 0.98 + 0.99 \times 0.01$
$= 0.0098 + 0.0099 = 0.0197$

(b) $P(\text{has it} \mid +) = \frac{0.01 \times 0.98}{0.0197}$

$= 0.497$

Answer: (a) 0.0197 (b) 0.497

9 Line and curve intersection (diagram) (All boards)

The diagram shows the curve $y = x^2 + 3x - 3$ and the line $y = 2x + 3$, meeting at A and B.
(a) Find the coordinates of A and B.
(b) Find the coordinates of the midpoint of AB.

$x^2 + 3x - 3 = 2x + 3$: $x^2 + x - 6 = 0$
$(x + 3)(x - 2) = 0$, $x = -3$ or 2
A$(-3, -3)$, B$(2, 7)$
Midpoint: average the coordinates

Answer: (a) A$(-3, -3)$, B$(2, 7)$ (b) $(-\frac{1}{2}, 2)$

10 Tangent through the origin (All boards)

The tangent to the curve $y = \ln(4x)$ at the point P passes through the origin.
Find the exact coordinates of P and the equation of the tangent.

$\frac{dy}{dx} = \frac{1}{x}$; tangent at $x = a$: $y - \ln(4a) = \frac{1}{a}(x - a)$

Through $(0, 0)$: $-\ln(4a) = -1$, so $4a = e$ and $a = \frac{e}{4}$

P $= (\frac{e}{4}, 1)$, gradient $\frac{1}{a} = 4e^{-1}$, so the tangent is $y = 4e^{-1}x$

Answer: P$(\frac{e}{4}, 1)$; $y = 4e^{-1}x$