**Day 354** **A Level Maths: A Tier 10-a-day: Model Solutions**

## 1 Conditional binomial probability (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X ~ B(10, 0.3). Find P(X = 5 | X ≥ 4) to 3 significant figures.

$P(X = 5 \mid X \geq 4) = \frac{P(X = 5)}{P(X \geq 4)}$ (since X = 5 implies X ≥ 4)

P(X = 5) = 0.103

P(X ≥ 4) = 1 − P(X ≤ 3) = 0.350

Ratio = 0.294

**Answer: 0.294**

## 2 Sum of multiples in a range (All boards)

Find the sum of all the integers from 1 to 117 inclusive that are not multiples of 4.

Sum of 1 to 117 = $\frac{117 \times 118}{2}$ = 6903

Multiples of 4 up to 117: 4, 8, ..., 116 (29 terms)

Their sum = $\frac{29}{2}(4 + 116)$ = 1740

Required sum = 6903 − 1740

**Answer: 5163**

## 3 Scalar product, magnitude of a sum (CIE (9709); Further Maths on other boards)

|a| = 6, |b| = 2 and the angle between a and b is 30°.

Find the exact values of a·b and |a + b|.

$\mathbf{a}\cdot\mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos 30° = 6 \times 2 \times \frac{\sqrt{3}}{2} = 6\sqrt{3}$

$|\mathbf{a} + \mathbf{b}|^2 = |\mathbf{a}|^2 + 2\mathbf{a}\cdot\mathbf{b} + |\mathbf{b}|^2 = 36 + 12\sqrt{3} + 4 = 12\sqrt{3} + 40$

$|\mathbf{a} + \mathbf{b}| = \sqrt{12\sqrt{3} + 40}$

**Answer: $\mathbf{a}\cdot\mathbf{b} = 6\sqrt{3}$, $|\mathbf{a} + \mathbf{b}| = \sqrt{12\sqrt{3} + 40}$**

## 4 Proof by exhaustion (AQA, Edexcel, OCR, MEI, Eduqas)

Prove that no cube number can be of the form 9k + 4, where k is an integer.

Cubes depend only on n mod 9: n = 0, 1, 2, 3, 4, 5, 6, 7, 8.

Remainders of $n^3$ on dividing by 9: 0, 1, 8, 0, 1, 8, 0, 1, 8

The only possible remainders are 0, 1, 8.

The remainder 4 never occurs.

**Answer: remainder 4 never occurs among cubes (mod 9)**

## 5 Newton-Raphson method (AQA, Edexcel, OCR, MEI, Eduqas)

$f(x) = x^2 - 13$. Show that the Newton-Raphson formula gives $x_{n+1} = \frac{1}{2}(x_n + \frac{13}{x_n})$,

and use it with $x_0 = 2$ to find $x_2$ as an approximation to $\sqrt{13}$, to 4 d.p.

$f'(x) = 2x$, so $x_{n+1} = x_n - \frac{x_n^2 - 13}{2x_n} = \frac{1}{2}(x_n + \frac{13}{x_n})$

$x_1 = \frac{1}{2}(2 + \frac{13}{2}) = \frac{17}{4}$

$x_2 = 3.6544$

**Answer: $x_2 = 3.6544$**

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## 6 Pulled at an angle on rough ground (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A 5 kg box is pulled along rough horizontal ground, $\mu = 0.25$, by a rope at angle $\theta$ above the horizontal, where $\tan\theta = \frac{5}{12}$. The tension is 60 N. Find (a) the normal reaction, (b) the acceleration of the box. Take $g = 9.8$. Give answers to 3 s.f. if not exact.

$\sin\theta = \frac{5}{13}$, $\cos\theta = \frac{12}{13}$

(a) Vertical: $R + 60\sin\theta = 5g$, so $R = \frac{337}{13}$

(b) Horizontal: $60\cos\theta - \mu R = 5a$

$\frac{720}{13} - 0.25\times\frac{337}{13} = 5a$

$a = \frac{2543}{260}$

**Answer: (a) 25.9 N (b) 9.78 m s$^{-2}$**

## 7 Combining two data sets (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The times in seconds of two groups: group X has n = 34, mean 57, sd 2; group Y has n = 17, mean 45, sd 5. Find the mean and standard deviation of all 51 values (3 s.f. for the sd).

$\Sigma x = 34 \times 57 + 17 \times 45 = 2703$, mean = 53

$\Sigma x^2$ (A) $= 34(2^2 + 57^2) = 110602$

$\Sigma x^2$ (B) $= 17(5^2 + 45^2) = 34850$

Variance $= \frac{145452}{51} - 53^2 = 43$, so sd = 6.56

**Answer: mean 53, sd 6.56**

## 8 Acceleration function of time (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves on the x-axis. At time t s its acceleration is $(6t - 12)$ m s$^{-2}$. When t = 0 it is at O with velocity $-8$ m s$^{-1}$. Find (a) its velocity when the acceleration is zero, (b) its displacement from O then.

$a = 0$ when $t = 2$

$v = \int a\,dt = 3t^2 - 12t - 8$ (using $v = -8$ when $t = 0$)

$v(2) = -20$

$x = \int v\,dt = t^3 - 6t^2 - 8t$ ($x = 0$ when $t = 0$)

$x(2) = -32$

**Answer: (a) $-20$ m s$^{-1}$ (b) $-32$ m**

## 9 Recurrence relations: limits (All boards)

$u_1 = 10$ and $u_{n+1} = -\frac{1}{2}u_n + 9$.

(a) Find the limit L of the sequence as $n \to \infty$.

(b) Find the least n for which $|u_n - L| < 0.1$.

(a) $L = -\frac{1}{2}L + 9$ gives $L = \frac{9}{1 - (-\frac{1}{2})} = 6$

(b) $u_n - L = (u_1 - L)\,r^{n-1} = 4 \times (-\frac{1}{2})^{n-1}$

$|4| \times \frac{1}{2}^{n-1} < \frac{1}{10}$

Check: n = 6 gives 0.12500, n = 7 gives 0.06250

**Answer: (a) L = 6 (b) n = 7**

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## 10 Connected rates of change: cones and bowls (AQA, Edexcel, OCR, MEI, Eduqas)

Water pours into a cone, vertex down, at 10 $cm^3s^{-1}$. The surface radius is always $\frac{h}{4}$.

Find the rate at which the depth is rising when h = 5 cm.

$V = \frac{1}{3}\pi(h/4)^2h = \frac{\pi h^3}{48}$, so $\frac{dV}{dh} = \frac{\pi h^2}{16}$

$\frac{dh}{dt} = \frac{dV}{dt \div dV/dh} = \frac{10 \times 16}{\pi h^2}$

h = 5: $\frac{dh}{dt} = \frac{32}{5\pi}$ cm $s^{-1}$

**Answer: $\frac{32}{5\pi}$ cm/s**