Day 353 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Implicit differentiation: horizontal tangents (All boards)

A curve has equation $4x^2 + 2xy + 2y^2 = 28$.
Find the coordinates of the points where the tangent is parallel to the x-axis.

$\frac{dy}{dx} = -\frac{8x + 2y}{2x + 4y}$

Horizontal tangent: $8x + 2y = 0$, so $y = -4x$

Substitute: $28x^2 = 28$, so $x^2 = 1$ and $x = \pm 1$

Points: $(1, -4)$ and $(-1, 4)$

Answer: $(1, -4)$ and $(-1, 4)$

2 Comparing data sets from the large data set (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Summary statistics for the daily mean windspeed (knots) in the same month are:
Heathrow: mean 28.5, standard deviation 4.1;
Leuchars: mean 11.5, standard deviation 5.1.
Make two comparisons.

Means: Heathrow has the higher mean (28.5 against 11.5), so on average the mean windspeed is greater at Heathrow

Standard deviations: Leuchars is larger at Leuchars (5.1 against 4.1)

so the readings at Leuchars are more variable and those at Heathrow are more consistent

Answer: Heathrow higher on average; Leuchars more variable

3 Dividing a line in a ratio (2D position vectors) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A and B have position vectors $\begin{pmatrix}4\\5\end{pmatrix}$ and $\begin{pmatrix}-2\\2\end{pmatrix}$. The point P lies on AB with AP : PB = 2 : 1.

Find the position vector of P and the exact distance OP.

$\text{OP} = \text{OA} + \frac{2}{3}\text{AB} = \begin{pmatrix}4\\5\end{pmatrix} + \frac{2}{3}\begin{pmatrix}-6\\-3\end{pmatrix}$

$= \begin{pmatrix}0\\3\end{pmatrix}$

$\text{OP} = \sqrt{0^2 + 3^2} = \sqrt{9} = 3$

Answer: $\begin{pmatrix}\mathbf{0}\\\mathbf{3}\end{pmatrix}$, OP = 3

4 Equation of a cubic from its graph (All boards)

The diagram shows a sketch of the curve y = f(x), where f(x) is a cubic.
Find f(x), giving your answer in expanded form.

Roots at $x = 2, -2, 3$, so $f(x) = k(x - 2)(x + 2)(x - 3)$

$f(0) = -24$: $k(-2)(2)(-3) = -24$

$k = -2$, then expand

Answer: $f(x) = -2x^3 + 6x^2 + 8x - 24$

5 Trapezium rule with over or under estimate (AQA, Edexcel, OCR, MEI, Eduqas)

Use the trapezium rule with 3 strips to estimate $\int_1^7 \sqrt{x + 3}\, dx$, giving your answer to 3 s.f.

State, with a reason, whether your estimate is an overestimate or an underestimate.

h = 2; y-values at x = 1, 3, 5, 7: 2.0000, 2.4495, 2.8284, 3.1623

T = 1[2.0000 + 3.1623 + 2(2.4495 + 2.8284)]

T = 15.7

The curve is concave (y'' < 0) on this interval, so the chords lie below the curve: underestimate

Answer: 15.7, an underestimate

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6 Rough incline: find the coefficient of friction (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle slides from rest down a rough plane inclined at α to the horizontal, where $\tan\alpha = \frac{3}{4}$. It moves 49 m in 5 s. Find (a) the coefficient of friction, (b) the speed after 5 s. Take g = 9.8.

$s = \frac{1}{2}at^2$: $49 = \frac{1}{2}\times a\times 5^2$, so $a = 3.92$
Along the slope: $mg\sin\alpha - \mu R = ma$, with $R = mg\cos\alpha$
$g(\sin\alpha - \mu\cos\alpha) = a$: $9.8(0.6 - 0.8\mu) = 3.92$
$0.6 - 0.8\mu = 0.4$, so $\mu = 0.25$
$v = at = 3.92\times 5$
Answer: $\mu = 0.25$; 19.6 m s^{-1}

7 Normal approximation to the Poisson (CIE)

X ~ Po(40). Use a suitable approximation to find P(X ≥ 38), to 3 significant figures.

$\lambda > 15$, so Y ~ N(40, 40) approximately
Continuity correction: P(Y > 37.5)
$z = -0.395$, so probability = 0.654
Answer: 0.654

8 Completing the square (All boards)

$f(x) = x^2 + 6x + 2$.

(a) Write f(x) in the form $a(x + p)^2 + q$.
(b) State the minimum value of f(x) and the value of x at which it occurs.
(c) Solve f(x) = 0, giving surds in simplest form.

$f(x) = (x + 3)^2 - 7$
(b) minimum value -7 when $x = -3$
(c) $(x + 3)^2 = 7$, so $(x + 3)^2 = 7$
$x + 3 = \pm\sqrt{7}$
Answer: (a) $(x + 3)^2 - 7$ (b) minimum -7 at $x = -3$ (c) $x = -3 \pm \sqrt{7}$

9 Closest approach of two particles (Edexcel, OCR, MEI)

At t = 0, P and Q have position vectors $(2\mathbf{i} - 4\mathbf{j})$ m and $(\mathbf{i} + 3\mathbf{j})$ m and velocities $(-6\mathbf{i} + 5\mathbf{j})$ m s^{-1} and $(-2\mathbf{i} + 2\mathbf{j})$ m s^{-1}. Find (a) the time when they are closest, (b) the least distance between them.

$\mathrm{PQ} = \mathbf{r}_P - \mathbf{r}_Q = (\mathbf{i} - 7\mathbf{j}) + t(-4\mathbf{i} + 3\mathbf{j})$
$d^2 = (-4t + 1)^2 + (3t - 7)^2 = 25t^2 - 50t + 50$
(a) $d(d^2)/dt = 0$: $50t + -50 = 0$, so $t = 1$
(b) At t = 1: $\mathrm{PQ} = -3\mathbf{i} - 4\mathbf{j}$, so $d = \sqrt{25}$
Answer: (a) t = 1 s (b) 5 m

10 Circle and line: chord length (AQA, Edexcel, OCR, MEI, Eduqas)

The line $4x - 3y = -5$ meets the circle $(x + 3)^2 + (y - 2)^2 = 36$ at the points A and B. Find the exact length of AB.

Centre $(-3, 2)$, radius 6. Distance from centre to line:

$$d = \frac{|-13|}{5} = \frac{13}{5}$$

$$\text{Half-chord}^2 = 36 - \left(\frac{13}{5}\right)^2 = \frac{731}{25}$$

$$\mathrm{AB} = 2\sqrt{\frac{731}{25}}$$

Answer: $\frac{10\sqrt{731}}{25}$