**Day 352** **A Level Maths: A Tier 10-a-day: Model Solutions**

## 1 Exact definite integrals of standard functions (All boards)

Find the exact value of

$\int_0^1 (2x - 1)^4 \, dx$.

Integrate: $[\frac{(2x - 1)^5}{10}]$

Substitute the limits and subtract: $\frac{1}{5}$

**Answer:** $\mathbf{\frac{1}{5}}$

## 2 Sum of multiples in a range (All boards)

Find the sum of all the multiples of 6 that lie strictly between 132 and 420.

First multiple 138, last multiple 414, common difference 6.

Number of terms n = $\frac{414 - 138}{6} + 1 = 47$

S = $\frac{47}{2}(138 + 414) = 12972$

**Answer: 12972**

## 3 Chain rule and gradients (All boards)

$y = \cos^2(2x)$. Find $\frac{dy}{dx}$ and its exact value at $x = \frac{\pi}{8}$.

Let $u = \cos(2x)$: $y = u^2$, so $\frac{dy}{dx} = 2u^1 \times \frac{du}{dx}$

$\frac{du}{dx} = -2\sin(2x)$, giving $\frac{dy}{dx} = -4\cos(2x)\sin(2x)$

At $x = \frac{\pi}{8}$: $\cos(2x) = \frac{\sqrt{2}}{2}$ and $\sin(2x) = \frac{\sqrt{2}}{2}$, so gradient $= -2$

**Answer:** $\mathbf{\frac{dy}{dx} = -4\cos(2x)\sin(2x)}$**; gradient** $\mathbf{= -2}$

## 4 Constant speed against friction (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A crate of mass 8 kg is pushed at constant speed along rough horizontal ground, $\mu = 0.3$, by a force of P N at angle $\theta$ below the horizontal, $\tan\theta = \frac{12}{5}$. Find P and the normal reaction. Take $g = 9.8$.

Constant speed, so the resultant force is zero. $\sin\theta = \frac{12}{13}$, $\cos\theta = \frac{5}{13}$

Vertical: $R = 8g + P\sin\theta$

Horizontal: $P\cos\theta = \mu R = 0.3(8g + P\sin\theta)$

$P(\frac{5}{13} - \frac{18}{65}) = 0.3 \times 8g$

$P = 218$, $R = 280$

**Answer: P = 218.4 N, R = 280 N**

## 5 Circle and line: tangent condition (AQA, Edexcel, OCR, MEI, Eduqas)

The line $-3x + 4y = k$ is a tangent to the circle

$(x + 1)^2 + (y - 3)^2 = 36$.

Find the two possible values of k.

Centre $(-1, 3)$, radius 6

Distance from centre to the line = radius:

$\frac{|15 - k|}{5} = 6$

$|15 - k| = 30$

**Answer: k = −15 or k = 45**

## 6 Interpreting a regression line (AQA, Edexcel, OCR, MEI, Eduqas)

The diagram shows data and the regression line y = 20 + 2.5x.
(a) Interpret the gradient.
(b) Estimate y when x = 11 and comment.

(a) On average, y increases by 2.5 marks for each extra 1 hours of x
(b) y = 20 + 2.5 × 11 = 47.5
x = 11 is within the range of the data (2 to 12): interpolation, so reliable
**Answer: (a) y increases by 2.5 marks per hours (b) 47.5, reliable**

## 7 Equations with cosec x and cot x (All boards)

Solve $\text{cosec}^2 x - 4\cot x + 2 = 0$
for $0° < x < 360°$. Give answers to 1 d.p.

Use $\text{cosec}^2 x \equiv 1 + \cot^2 x$: $\cot^2 x - 4\cot x + 3 = 0$
$\cot x = 1$ or $\cot x = 3$
$\cot x = 1$ means $\tan x = 1$: $x = 45.0°, 225.0°$
$\cot x = 3$ means $\tan x = \frac{1}{3}$: $x = 18.4°, 198.4°$
**Answer: $x = 18.4°, 45.0°, 198.4°, 225.0°$**

## 8 Proof by contradiction (AQA, Edexcel, OCR, MEI, Eduqas)

Prove by contradiction that $\sqrt{3}$ is irrational.

Assume $\sqrt{3} = \frac{a}{b}$ with a, b integers with no common factor.

Then $3b^2 = a^2$, so $a^2$ is a multiple of 3, hence a is a multiple of 3.
Write $a = 3c$. Then $3b^2 = 9c^2$, so $b^2 = 3c^2$.
So b is also a multiple of 3, contradicting no common factor.
**Answer: contradiction, so $\sqrt{3}$ is irrational**

## 9 Position vector and velocity (AQA, Edexcel, OCR, MEI, Eduqas)

A particle P has position vector $\mathbf{r} = (t^3 + 6t)\mathbf{i} + (t^2 - 6t)\mathbf{j}$ metres at time t s. Find (a) the speed of P when t = 2, (b) the value of t when P is moving parallel to $\mathbf{i}$. Give answers to 3 s.f. if not exact.

$\mathbf{v} = d\mathbf{r}/dt = (3t^2 + 6)\mathbf{i} + (2t - 6)\mathbf{j}$
(a) t = 2: $\mathbf{v} = 18\mathbf{i} - 2\mathbf{j}$
Speed $= \sqrt{(18^2 + -2^2)} = \sqrt{328}$
(b) Parallel to $\mathbf{i}$ means the $\mathbf{j}$ component is zero: $2t - 6 = 0$, so t = 3
**Answer: (a) 18.1 m s$^{-1}$ (b) t = 3**

## 10 Conditional probability without replacement (All boards)

A bag has 5 black and 3 white counters. Two are taken at random without replacement.
Given that the second counter is black, find the probability that the first was black.

$\text{P(black then black)} = \frac{5}{8} \times \frac{4}{7} = \frac{5}{14}$

$\text{P(white then black)} = \frac{3}{8} \times \frac{5}{7} = \frac{15}{56}$

$\text{P(second black)} = \frac{5}{14} + \frac{15}{56} = \frac{5}{8}$

$$\text{P(first black | second black)} = \frac{\frac{5}{14}}{\frac{5}{8}}$$

$= \frac{4}{7}$

**Answer: $\frac{4}{7}$**