Day 351 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Finding an unknown standard deviation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X is normally distributed with mean μ and standard deviation 8.
Given that $P(X < 61) = 0.0228$, find μ to 3 significant figures.

$P(Z < z) = 0.0228$, so $z = -2.00$

$\frac{61 - \mu}{8} = -2.00$

$\mu = 61 - (-2.00)(8) = 77.0$

Answer: $\mu = 77.0$

2 Exact definite integrals of standard functions (All boards)

Find the exact value of

$\int_0^3 (4x)^2 \, dx$.

Integrate: $[\frac{(4x)^3}{12}]$

Substitute the limits and subtract: 144

Answer: 144

3 Selections with combinations (OCR, MEI, Eduqas, CIE)

A group of 6 boys and 8 girls. A team of 3 is chosen at random.
Find the probability that all the team are of the same sex.

Total teams = C(14, 3) = 364

Same sex: C(6, 3) + C(8, 3) = 20 + 56 = 76

$P = \frac{76}{364}$

$= \frac{19}{91}$

Answer: $\frac{19}{91}$

4 Arithmetic modelling (All boards)

A charity raises £500 in its first year and in each later year raises £50 more than in the previous year.
(a) How much does it raise in year 17? (b) In which year does the total raised first exceed £5000?

(a) $u_{17} = 500 + 16 \times 50 = 1300$

(b) $S_m = \frac{m}{2}(1000 + (m - 1) \times 50) > 5000$

$S_7 = 4550 < 5000$ and $S_8 = 5400 > 5000$

Answer: (a) £1300 (b) year 8

5 Scalar product, magnitude of a sum (CIE (9709); Further Maths on other boards)

$|a| = 5$, $|b| = 4$ and the angle between a and b is 120°.
Find the exact values of $a \cdot b$ and $|a + b|$.

$a \cdot b = |a||b|\cos 120° = 5 \times 4 \times -\frac{1}{2} = -10$

$|a + b|^2 = |a|^2 + 2a \cdot b + |b|^2 = 25 + -20 + 16 = 21$

$|a + b| = \sqrt{21}$

Answer: $a \cdot b = -10$, $|a + b| = \sqrt{21}$

6 Ships on collision courses (vectors) (AQA, Edexcel, OCR, MEI, Eduqas)

At 12:00 ship P has position $(16\mathbf{i} - 20\mathbf{j})$ km and velocity $(-4\mathbf{i} + 5\mathbf{j})$ km h^{-1}; ship Q has position $(16\mathbf{i} + 20\mathbf{j})$ km and velocity $(-4\mathbf{i} - 3\mathbf{j})$ km h^{-1}. Show that the ships collide and find the time and position vector of the collision.

At time t hours: $\mathbf{r}_P = (16\mathbf{i} - 20\mathbf{j}) + t(-4\mathbf{i} + 5\mathbf{j})$, $\mathbf{r}_Q = (16\mathbf{i} + 20\mathbf{j}) + t(-4\mathbf{i} - 3\mathbf{j})$

$\mathbf{j}$: $-20 + 5t = 20 + -3t$, so $t = 5$

$\mathbf{i}$: $16 + 5\times(-4) = -4$

Check with Q: $\mathbf{i}$: $16 + 5\times(-4) = -4$, $\mathbf{j}$: $20 + 5\times(-3) = 5$

Both components agree at $t = 5$, so they collide at $(-4\mathbf{i} + 5\mathbf{j})$ km

Answer: t = 5 hours (at 17:00); r = (−4i + 5j) km

7 Reducing a relationship to a straight line (graph) (All boards)

The diagram shows the graph of ln y against ln x for $y = A x^n$.
Find the value of n and the exact value of A.

ln y = ln A + n ln x: gradient n, intercept ln A

$n = \frac{12 - 8}{4 - 2} = 2$

At (2, 8): $\ln A = 8 - 2 \times 2 = 4$, so $A = e^4$

Answer: n = 2, A = e^4

8 Sum to infinity: two possible ratios (All boards)

The second term of a geometric series is 9 and its sum to infinity is 48.
Find the two possible values of the first term.

$ar = 9$ and $\frac{a}{(1 - r)} = 48$

$a = 48(1 - r)$, so $48(1 - r)\, r = 9$

$48r^2 - 48r + 9 = 0$: $r = \frac{1}{4}$ or $r = \frac{3}{4}$

$a = 48(1 - r) = 36$ or 12

Answer: a = 12 or a = 36

9 Periodic sequences (All boards)

A sequence is defined by $u_1 = 5$ and $u_{n+1} = \frac{1}{1 - u_n}$.

(a) Find u_2 and u_3.

(b) Find u_{100} and $\sum_{r=1}^{100} u_r$.

$u_2 = -\frac{1}{4}$, $u_3 = \frac{4}{5}$, $u_4 = 5$

$u_4 = u_1$, so the sequence repeats with period 3.

$100 = 3 \times 33 + 1$, so $u_{100} = u_1 = 5$

Sum of one period = $\frac{111}{20}$; 33 full periods plus 1 extra term(s)

Answer: (a) $-\frac{1}{4}$, $\frac{4}{5}$ (b) $u_{100} = 5$, sum = $\frac{3763}{20}$

10 Projected up a rough incline (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle is projected at 9.8 m s^{-1} up a line of greatest slope of a rough plane inclined at α, where $\tan\alpha = \frac{3}{4}$. The coefficient of friction is 0.5. Find (a) the greatest distance it moves up the plane, (b) its speed on returning to the start. Take g = 9.8. Give answers to 3 s.f. if not exact.

Up: deceleration = $g(\sin\alpha + \mu\cos\alpha) = 9.8(0.6 + 0.5\times0.8) = 9.8$

(a) $0 = 9.8^2 - 2\times9.8\times d$, so $d = 4.9$

Down: $\tan\alpha = \frac{3}{4} > \mu$, so it slides back; acceleration = $g(\sin\alpha - \mu\cos\alpha) = 1.96$

(b) $v^2 = 2\times1.96\times4.9 = 19.208$

$v = \sqrt{19.208}$

Answer: (a) 4.9 m (b) 4.38 m s^{-1}