Day 350 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Arithmetic sequences: finding a and d (All boards)

The 6th term of an arithmetic sequence is 0 and the 12th term is −12.
Find the first term and common difference, and the sum of the first 25 terms.

$u_{12} - u_6 = 6\,d = -12 - 0$ so $d = -2$

$a + 5\,d = 0$ so $a = 10$

$S_{25} = \frac{25}{2}(2 \times 10 + 24 \times (-2))$

Answer: a = 10, d = −2, S_{25} = −350

2 Lift and cable tension (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A lift of mass 200 kg carries a person of mass 75 kg. The cable tension is 2777.5 N and the lift is accelerating. Find (a) the magnitude and direction of the acceleration, (b) the force of the floor on the person. Take g = 9.8.

Whole system (mass 275 kg), taking upwards as positive: $T - 275\,g = 275\,a$

$2777.5 - 2695 = 275\,a$, so $a = 0.3$

(a) a is positive, so the acceleration is 0.3 m s^{-2} upwards

(b) Person: $R - 75\,g = 75\,a$, so $R = 75(9.8 + 0.3)$

Answer: (a) 0.3 m s^{-2} upwards (b) 757.5 N

3 Proving trigonometric identities (All boards)

Prove the identity

$\sec^2 2x + \operatorname{cosec}^2 2x \equiv 4\operatorname{cosec}^2 4x$

$$\text{LHS} = \frac{1}{\cos^2 2x} + \frac{1}{\sin^2 2x} = \frac{\sin^2 2x + \cos^2 2x}{\sin^2 2x \cos^2 2x} = \frac{1}{\sin^2 2x \cos^2 2x}$$

$$= \frac{4}{(2\sin 2x \cos 2x)^2} = \frac{4}{\sin^2 4x} = \text{RHS}$$

Answer: Proved

4 Circle with a given diameter (AQA, Edexcel, OCR, MEI, Eduqas)

A(7, −2) and B(−3, −2) are the ends of a diameter of a circle.
(a) Find the equation of the circle, in the form $x^2 + y^2 + ax + by + c = 0$.
(b) Show that C(−1, 2) lies on the circle.

Centre = midpoint of AB = (2, −2)

$r^2 = (5)^2 + (0)^2 = 25$

$(x - 2)^2 + (y + 2)^2 = 25$: $x^2 + y^2 - 4x + 4y - 17 = 0$

(b) $CM^2 = (-3)^2 + (4)^2 = 25 = r^2$

Answer: (a) $x^2 + y^2 - 4x + 4y - 17 = 0$

5 Ladder in limiting equilibrium (AQA, Edexcel, OCR, MEI)

A uniform ladder AB (15 m, 10 kg) rests with A on rough ground, μ = 0.3, and B against a smooth wall, B being 12 m above the ground. A man of mass 60 kg climbs the ladder. How far along the ladder from A can he climb before it slips?

Foot of ladder is $\sqrt{(15^2 - 12^2)} = 9$ m from the wall

Vertical: $R = (10 + 60)g$. Limiting friction: $F = \mu R = 0.3(10 + 60)g$

Horizontal: S = F (wall reaction S)

Moments about A (man at distance x along ladder, horizontal distance x×9/15):

$S \times 12 = 10\,g \times 4.5 + 60\,g \times (9\,x \div 15)$

$21 \times 12 = 45 + 36\,x$ (g cancels), so $x = 5.75$

Answer: 5.75 m

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6 Exponential equations: quadratic in b^x (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $2 \times 2^{2x} - 18 \times 2^x + 16 = 0$.

Let $y = 2^x$. Then $2y^2 - 18y + 16 = 0$
$2(y - 8)(y - 1) = 0$
$y = 8$ or $y = 1$
$2^x = 8$ gives $x = 3$; $2^x = 1$ gives $x = 0$
Answer: $x = 0$ or $x = 3$

7 Three-set Venn diagram counts (All boards)

Of 42 students, 15 study Maths (M), 21 Physics (P) and 18 Chemistry (C).
7 study M and P, 8 study M and C, 8 study P and C, and 4 study all three.
One student is chosen at random. Given that a student does not study Maths, find the probability that they study Physics.

Venn regions: M only 4, P only 10, C only 6,
MP only 3, MC only 4, PC only 4, all three 4, outside all 7
Not Maths: $42 - 15 = 27$
Physics but not Maths: $21 - 7 = 14$
Probability = $\frac{14}{27}$
Answer: $\frac{14}{27}$

8 Reducing a relationship to a straight line (graph) (All boards)

The diagram shows the graph of lg y against lg x for $y = A x^n$.
Find the value of n and the value of A.

$\lg y = \lg A + n \lg x$: gradient n, intercept $\lg A$
$n = \frac{7 - 4}{2 - 1} = 3$
At (1, 4): $\lg A = 4 - 3 \times 1 = 1$, so $A = 10^1 = 10$
Answer: $n = 3$, $A = 10$

9 Continuous random variable: cumulative distribution and median (CIE)

The random variable X has pdf $f(x) = \frac{3}{8}x^2$ for $0 \leq x \leq 2$, and 0 otherwise.

(a) Find the cumulative distribution function F(x) for $0 \leq x \leq 2$.
(b) Find P(X > 1). (c) Find the median of X to 3 s.f.

(a) $F(x) = \int_0^x f(t)\,dt = \frac{x^3}{8}$
(b) $P(X > 1) = 1 - F(1) = \frac{7}{8}$
(c) $F(m) = \frac{1}{2}$: $m^3 = \frac{2^3}{2}$, so $m = 2 \times (0.5)^{\frac{1}{3}}$
$m = 1.59$
Answer: (b) $\frac{7}{8}$ (c) 1.59

10 Exact definite integrals of standard functions (All boards)

Find the exact value of

$\int_0^2 \frac{6}{2x + 1}\,dx$.

Integrate: $[\frac{6}{2}\ln(2x + 1)]$
Substitute the limits and subtract: $3 \ln 5$
Answer: $3 \ln 5$