Day 35 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Binomial expansion: a coefficient (All boards)

Find the coefficient of x^4 in the expansion of $(1 - 2x)^6$.

Term in x^4: ${}^6C_4 \times 1^2 \times (-2x)^4$

$= 15 \times 1 \times 16$

Answer: 240

2 Proving trigonometric identities (All boards)

Prove the identity

$\frac{\sin 2x}{1 - \cos 2x} \equiv \cot x$

$\text{LHS} = \frac{2\sin x \cos x}{1 - (1 - 2\sin^2 x)} = \frac{2\sin x \cos x}{2\sin^2 x}$

$= \frac{\cos x}{\sin x} = \cot x = \text{RHS}$

Answer: Proved

3 Algebraic fractions (All boards)

Simplify $\frac{x^2 - 1}{(2x - 1)} \div \frac{(x + 1)}{(2x - 4)}$.

$\frac{x^2 - 1}{(2x - 1)} \times \frac{(2x - 4)}{(x + 1)}$

$x^2 - 1 = (x - 1)(x + 1)$

Cancel $(x + 1)$.

Answer: $\mathbf{\frac{(x - 1)(2x - 4)}{(2x - 1)}}$

4 Equations with sin 2x and a trig function (All boards)

Solve $\sin 2\theta = \sqrt{3} \sin \theta$ for $0 \leq \theta \leq 2\pi$.

$\sin 2\theta \equiv 2\sin \theta \cos \theta$: $\sin \theta (2\cos \theta - \sqrt{3}) = 0$

$\sin \theta = 0$: $\theta = 0, \pi, 2\pi$

$\cos \theta = \frac{\sqrt{3}}{2}$: $\theta = \frac{\pi}{6}, \frac{11\pi}{6}$

Do not divide by $\sin \theta$, as that would lose solutions

Answer: $\boldsymbol{\theta = 0, \frac{\pi}{6}, \pi, \frac{11\pi}{6}, 2\pi}$

5 Vertical motion under gravity (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A ball is thrown vertically upwards at 9.8 m s^{-1} from a point h m above horizontal ground, and hits the ground 3 s later. Find (a) h, (b) the greatest height above the ground. Take g = 9.8.

(a) Take upwards as positive: $s = -h$, $u = 9.8$, $a = -9.8$, $t = 3$

$-h = 9.8 \times 3 - 4.9 \times 3^2$, so $h = 14.7$

(b) At the highest point $v = 0$: $0 = 9.8^2 - 2 \times 9.8 \times s$

$s = 4.9$ m above the point of projection

Greatest height above the ground = 14.7 + 4.9

Answer: (a) 14.7 m (b) 19.6 m

6 Bayes' theorem in context (All boards)

0.5% of people have a disease. The screening test is positive for 96% of those who have it and for 4% of those who do not.
(a) Find the probability that a person tests positive.
(b) Given a positive test, find the probability that the person has the disease (3 s.f.).

(a) $P(+) = 0.005 \times 0.96 + 0.995 \times 0.04$
$= 0.0048 + 0.0398 = 0.0446$

(b) $P(\text{has it} \mid +) = \dfrac{0.005 \times 0.96}{0.0446}$

$= 0.108$

Answer: (a) 0.0446 (b) 0.108

7 Velocity, rest and total distance (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle P moves on the x-axis with velocity $v = (6t^2 - 36t + 30)$ m s^{-1} at time t s, $t \geq 0$, and starts at O. Find (a) the times when P is at instantaneous rest, (b) the total distance P travels in the first 6 s.

(a) $v = 6(t - 1)(t - 5) = 0$, so $t = 1$ or $t = 5$

(b) Displacement $x = \int v\,dt = 2t^3 - 18t^2 + 30t$
$x(1) = 14$, $x(5) = -50$, $x(6) = -36$
Direction changes at $t = 1$ and $t = 5$, so distance $= |14| + |-50 - 14| + |-36 - (-50)|$

Answer: (a) $t = 1$, $t = 5$ (b) 92 m

8 Geometric sequences: finding a and r (All boards)

The 4th term of a geometric sequence is -96 and the 6th term is -384.
Find the common ratio and the first term.
Find the sum of the first 7 terms.

$r^2 = \dfrac{-384}{-96} = 4$ so $r = -2$

$a r^3 = -96$ so $a = 12$

$S_7 = \dfrac{a(r^7 - 1)}{(r - 1)} = \dfrac{12((-128) - 1)}{-3}$

Answer: $r = -2$, $a = 12$, $S_7 = 516$

9 Greatest and least values using R-form (All boards)

$g(x) = \dfrac{1}{30 + 20 \sin x + 21 \cos x}$

(a) Write $20 \sin x + 21 \cos x$ in the form $R \sin(x + \alpha)$, with α acute.
(b) Find the greatest and least values of g(x) and the smallest positive x at which the greatest value occurs (1 d.p.).

$R = \sqrt{20^2 + 21^2} = 29$, $\alpha = \tan^{-1}\dfrac{21}{20} = 46.4°$

Denominator $= 30 + 29 \sin(x + 46.4°)$, ranging from 1 to 59

Greatest $g = \dfrac{1}{1}$ (denominator least), least $g = \dfrac{1}{59}$

Least denominator when $\sin(x + 46.4°) = -1$: $x + 46.4° = 270°$
$x = 223.6°$

Answer: $R = 29$, $\alpha = 46.4°$; greatest $\frac{1}{1}$, least $\frac{1}{59}$; $x = 223.6°$

10 Hypothesis test for a Poisson mean (CIE)

Complaints occur as a Poisson process at a mean rate of 2 per day.
After a change, the number recorded in one day is 4.
Test at the 5% level whether the mean rate has increased.

$H_0: \lambda = 2$, $H_1: \lambda > 2$; $X \sim \text{Po}(2)$
$P(X \geq 4) = 1 - P(X \leq 3) = 0.143$
$0.143 > 0.05$
Do not reject H_0: insufficient evidence of a change.

Answer: p-value 0.143; do not reject H_0