Day 349 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Equations with sin 2x and a trig function (All boards)

Solve $\sin 2\theta = -\sin\theta$ for $0° \le \theta \le 360°$.

$\sin 2\theta \equiv 2\sin\theta\cos\theta$: $\sin\theta(2\cos\theta + 1) = 0$
$\sin\theta = 0$: $\theta = 0°, 180°, 360°$

$\cos\theta = -\frac{1}{2}$: $\theta = 120°, 240°$

Do not divide by $\sin\theta$, as that would lose solutions

Answer: $\theta = 0°, 120°, 180°, 240°, 360°$

2 Speed-time graphs (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the speed-time graph for a train journey of 1750 m. Find (a) V, (b) the deceleration in the final stage.

(a) Distance = area of trapezium = $\frac{1}{2}(90 + 50) \times V$
$1750 = 70V$, so $V = 25$
(b) Deceleration = $V \div 20 = 25 \div 20$

Answer: (a) V = 25 (b) 1.25 m s^{-2}

3 Circle with a given diameter (AQA, Edexcel, OCR, MEI, Eduqas)

A(−10, −8) and B(6, 4) are the ends of a diameter of a circle.
(a) Find the equation of the circle, in the form $x^2 + y^2 + ax + by + c = 0$.
(b) Show that C(4, −10) lies on the circle.

Centre = midpoint of AB = (−2, −2)
$r^2 = (-8)^2 + (-6)^2 = 100$
$(x + 2)^2 + (y + 2)^2 = 100$: $x^2 + y^2 + 4x + 4y - 92 = 0$
(b) $CM^2 = (6)^2 + (-8)^2 = 100 = r^2$

Answer: (a) $x^2 + y^2 + 4x + 4y - 92 = 0$

4 Poisson probabilities (CIE)

Flaws occur in a metre of cable at a mean rate of 1.5 per metre, as a Poisson process.
Find the probability that in one such period there are
(a) exactly 4 (b) at least 4 (c) fewer than 4 (3 s.f.).

$X \sim Po(1.5)$
(a) $P(X = 4) = e^{-1.5}\,1.5^4/24! = 0.0471$
(b) $1 - P(X \le 3) = 1 - 0.934 = 0.0656$
(c) $P(X \le 3) = 0.934$

Answer: (a) 0.0471 (b) 0.0656 (c) 0.934

5 Binomial expansion: (a + bx)^n for rational n (All boards)

Expand $(8 + x)^{\frac{1}{3}}$ in ascending powers of x, up to and including x^2.
State the range of x for the expansion is valid.

$(8 + x)^{\frac{1}{3}} = 8^{\frac{1}{3}}(1 + \frac{1x}{8})^{\frac{1}{3}}$

$8^{\frac{1}{3}} = 2$

Expand $(1 + y)^{\frac{1}{3}}$ with $y = \frac{1x}{8}$

Valid for $|\frac{1x}{8}| < 1$, i.e. $|x| < 8$

Answer: $2 + \frac{1}{12}x - \frac{1}{288}x^2$; $|x| < 8$

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6 Coding mean and standard deviation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

For 40 values of x, the coded variable $y = \frac{x - 50}{4}$ gives

$\Sigma y = 40$ and $\Sigma y^2 = 290$.

Find the mean and standard deviation of x.

$\bar{y} = \frac{40}{40} = 1$

$\sigma_y^2 = \frac{290}{40} - 1^2 = 6.25$, so $\sigma_y = 2.5$

$x = 4y + 50$, so mean $= 4 \times 1 + 50 = 54$

Standard deviation $= 4 \times 2.5 = 10$

Answer: mean 54, sd 10

7 Connected rates of change: cones and bowls (AQA, Edexcel, OCR, MEI, Eduqas)

A hemispherical bowl of radius 8 cm is filled at 20 $cm^3 s^{-1}$. When the depth is h, $V = \pi(8h^2 - \frac{1}{3}h^3)$.

Find $\frac{dh}{dt}$ when h = 3 cm.

$\frac{dV}{dh} = \pi(16h - h^2) = \pi h(16 - h)$

$\frac{dh}{dt} = \frac{20}{\pi h(16 - h)}$

$h = 3$: $\frac{dh}{dt} = \frac{20}{\pi \times 3 \times 13} = \frac{20}{39\pi}$ cm s^{-1}

Answer: $\frac{20}{39\pi}$ cm/s

8 Rough incline: find the coefficient of friction (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle slides from rest down a rough plane inclined at α to the horizontal, where $\tan\alpha = \frac{5}{12}$. It moves 8.82 m in 3 s.

Find (a) the coefficient of friction, (b) the speed after 3 s. Take g = 9.8.

$s = \frac{1}{2}at^2$: $8.82 = \frac{1}{2} \times a \times 3^2$, so $a = 1.96$

Along the slope: $mg\sin\alpha - \mu R = ma$, with $R = mg\cos\alpha$

$g(\sin\alpha - \mu\cos\alpha) = a$: $9.8(\frac{5}{13} - \frac{12}{13}\mu) = 1.96$

$\frac{5}{13} - \frac{12}{13}\mu = 0.2$, so $\mu = 0.2$

$v = at = 1.96 \times 3$

Answer: $\mu = 0.2$; 5.88 m s^{-1}

9 Tangent through the origin (All boards)

The line through the origin is a tangent to the curve $y = x^2 + 5$.

Find the gradients of the two possible tangents and the points of contact.

Tangent at x = a: $y - (a^2 + 5) = 2a(x - a)$. Through (0, 0): $-a^2 - 5 = -2a^2$

So $a^2 = 5$ and $a = \pm\sqrt{5}$

Gradient $2a = \pm 2\sqrt{5}$

Answer: gradients $\pm 2\sqrt{5}$ at points $(\pm\sqrt{5}, 10)$

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10 Partial fractions and binomial expansion (All boards)

$f(x) = \frac{5x - 3}{(1 - x)(1 - 3x)}$.

(a) Express f(x) in partial fractions.

(b) Hence expand f(x) up to x^2.

(c) State the range of x for which the expansion is valid.

(a) $f(x) = \frac{A}{(1 - x)} + \frac{B}{(1 - 3x)}$: $A = -1$, $B = -2$

(b) $-1(1 - x)^{-1} = -1(1 + x + 1x^2 ...)$

Add the series for both fractions.

(c) Need $|x| < 1$ and $|3x| < 1$

Answer: (a) $\frac{-1}{(1 - x)} + \frac{-2}{(1 - 3x)}$ **(b)** $-3 - 7x - 19x^2$ **(c)** $|x| < \frac{1}{3}$