Day 348 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Constant acceleration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A cyclist passes P with speed 6 m s^{-1} and accelerates uniformly. She travels 85 m in the next 10 s, reaching Q. Find (a) her acceleration, (b) her speed at Q, (c) her speed at the midpoint of PQ. Give answers to 3 s.f. if not exact.

(a) $s = ut + \frac{1}{2}at^2$: $85 = 6\times10 + \frac{1}{2}\times a\times10^2$, so $a = 0.5$
(b) $v = u + at = 6 + 0.5\times10 = 11$
(c) For the midpoint, $s = 42.5$: $v^2 = u^2 + 2as = 6^2 + 2\times0.5\times42.5$
$v^2 = 78.5$
Answer: (a) 0.5 m s^{-2} (b) 11 m s^{-1} (c) 8.86 m s^{-1}

2 Smallest whole number solving an exponential inequality (All boards)

£5000 is invested at 5% compound interest per year.
Find the smallest whole number of years for the value to exceed £10000.

$5000 \times 1.05^n > 10000$, so $1.05^n > 2$
$n \ln 1.05 > \ln 2$, so $n > \frac{\ln 2}{\ln 1.05} = 14.21$
$n = 15$
Answer: n = 15 years

3 Comparing data sets from the large data set (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Summary statistics for the daily mean visibility (Dm) in the same month are:
Perth: mean 26, standard deviation 1.6;
Jacksonville: mean 23, standard deviation 3.9.
Make two comparisons.

Means: Perth has the higher mean (26 against 23), so on average the mean visibility is greater at Perth
Standard deviations: Jacksonville is larger at Jacksonville (3.9 against 1.6)
so the readings at Jacksonville are more variable and those at Perth are more consistent
Answer: Perth higher on average; Jacksonville more variable

4 Forces in equilibrium with vectors (AQA, Edexcel, OCR, MEI, Eduqas)

Forces $F_1 = (3\mathbf{i} - 4\mathbf{j})$ N, $F_2 = (-12\mathbf{i} - 5\mathbf{j})$ N and $F_3 = (a\mathbf{i} + b\mathbf{j})$ N keep a 2 kg particle in equilibrium. Find (a) a and b. F_2 is then removed; (b) find the magnitude of the acceleration.

(a) Equilibrium: $F_1 + F_2 + F_3 = 0$
i: $3 + (-12) + a = 0$, so $a = 9$
j: $-4 + (-5) + b = 0$, so $b = 9$
(b) The resultant is now $F_1 + F_3 = -F_2$, of magnitude $\sqrt{(-12^2 + -5^2)} = 13$ N
$a = 13 \div 2$
Answer: (a) a = 9, b = 9 (b) 6.5 m s^{-2}

5 Finding an unknown standard deviation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X is normally distributed with mean μ and standard deviation 8.
Given that P(X < 38) = 0.3085, find μ to 3 significant figures.

$P(Z < z) = 0.3085$, so $z = -0.500$
$\frac{38 - \mu}{8} = -0.500$
$\mu = 38 - (-0.500)(8) = 42.0$
Answer: μ = 42.0

6 Inverse of a rational function (All boards)

$f(x) = \frac{3x+2}{3x-1}$, $x \in \mathbb{R}$, $x \neq \frac{1}{3}$.

Find $f^{-1}(x)$ and state its domain.

$y = \frac{3x+2}{3x-1}$: $y(3x-1) = 3x+2$

$3xy - y = 3x + 2$

$x(3y-3) = y + 2$

$x = \frac{y+2}{3y-3}$, then swap x and y

Answer: $f^{-1}(x) = \frac{x+2}{3x-3}$, $x \neq 1$

7 Optimisation problems (All boards)

A window is a rectangle of width 2x m and height h m, topped by a semicircle of radius x m. Its perimeter is 40 m.

Show that its area is $A = 40x - (2 + \frac{\pi}{2})x^2$, and find the maximum area to 3 s.f.

Perimeter: $2x + 2h + \pi x = 40$, so $h = \frac{40 - 2x - \pi x}{2}$

$A = 2xh + \frac{1}{2}\pi x^2 = x(40 - 2x - \pi x) + \frac{1}{2}\pi x^2 = 40x - (2 + \frac{\pi}{2})x^2$

$\frac{dA}{dx} = 40 - (4 + \pi)x = 0$, so $x = \frac{40}{4+\pi} = 5.60$

$\frac{d^2A}{dx^2} = -(4 + \pi) < 0$, so a maximum; $A = 112$

Answer: $A_{max} = 112\ m^2$ ($x = 5.60$)

8 Proof by contradiction (AQA, Edexcel, OCR, MEI, Eduqas)

Prove by contradiction that there are no integers m and n such that $6m + 15n = 118$.

Assume $6m + 15n = 118$ for integers m, n.
Then $3(2m + 5n) = 118$, so 3 divides 118.
But 118 is not a multiple of 3. Contradiction.
Answer: no such integers exist

9 Equations with cosec x and cot x (All boards)

Solve $2\text{cosec}^2\theta - 7\cot\theta + 1 = 0$
for $-180° < \theta < 180°$, $\theta \neq 0$. Give answers to 1 d.p.

Use $\text{cosec}^2\theta \equiv 1 + \cot^2\theta$: $2\cot^2\theta - 7\cot\theta + 3 = 0$

$\cot\theta = 3$ or $\cot\theta = \frac{1}{2}$

$\cot\theta = 3$ means $\tan\theta = \frac{1}{3}$: $\theta = -161.6°, 18.4°$

$\cot\theta = \frac{1}{2}$ means $\tan\theta = 2$: $\theta = -116.6°, 63.4°$

Answer: $\theta = -161.6°, -116.6°, 18.4°, 63.4°$

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10 Periodic sequences (All boards)

A sequence is defined by $u_1 = 3$ and $u_{n+1} = 1 - \frac{1}{u_n}$.

(a) Find u_2 and u_3.

(b) Find u_{150} and $\sum_{r=1}^{150} u_r$.

$u_2 = \frac{2}{3}$, $u_3 = -\frac{1}{2}$, $u_4 = 3$

$u_4 = u_1$, so the sequence repeats with period 3.

$150 = 3 \times 50 + 0$, so $u_{150} = u_3 = -\frac{1}{2}$

Sum of one period = $\frac{19}{6}$; 50 full periods plus 0 extra term(s)

Answer: (a) $\frac{2}{3}$, $-\frac{1}{2}$ (b) $u_{150} = -\frac{1}{2}$, sum = $\frac{475}{3}$