Day 347 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Parametric equations: Cartesian form (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has parametric equations $x = 4t + 3$, $y = 3t^2 + 3$.
Find a Cartesian equation of the curve in the form $y = f(x)$.

From x: $t = \frac{x - 3}{4}$

$y = 3(\frac{x - 3}{4})^2 + 3$

Expand.

Answer: $\mathbf{y = \frac{3}{16}x^2 - \frac{9}{8}x + \frac{75}{16}}$

2 Equilibrium on a slope: range of force (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A 5 kg particle rests on a rough plane inclined at α, $\tan\alpha = \frac{12}{5}$, $\mu = 0.1$. A force of P N acts up a line of greatest slope and the particle stays at rest. Find the range of values of P. Take $g = 9.8$. Give answers to 3 s.f. if not exact.

$\sin\alpha = \frac{12}{13}$, $\cos\alpha = \frac{5}{13}$, $R = 5g\cos\alpha = \frac{245}{13}$ N

Limiting friction $= \mu R = \frac{49}{26}$ N

P smallest: friction acts up the slope: $P + \mu R = 5g\sin\alpha$, $P = \frac{588}{13} - \frac{49}{26}$

P greatest: friction acts down the slope: $P = \frac{588}{13} + \frac{49}{26}$

Answer: 43.3 N ≤ P ≤ 47.1 N

3 Independent and mutually exclusive events (All boards)

$P(A) = 0.1$ and $P(A \cup B) = 0.64$.
(a) Given that A and B are independent, find P(B).
(b) Given instead that A and B are mutually exclusive, find P(B).
(c) In the case in (a), find $P(A' \cap B)$.

(a) $P(A \cup B) = P(A) + P(B) - P(A)P(B)$, so $0.64 = 0.1 + P(B)(1 - 0.1)$

$P(B) = \frac{0.64 - 0.1}{0.9} = 0.6$

(b) $P(A \cap B) = 0$, so $P(B) = 0.64 - 0.1 = 0.54$

(c) $P(A' \cap B) = P(A')P(B) = 0.9 \times 0.6 = 0.54$

Answer: (a) 0.6 (b) 0.54 (c) 0.54

4 Geometric sequences: finding a and r (All boards)

The 3rd term of a geometric sequence is 9 and the 5th term is 81.
Find the common ratio and the first term.
Find the sum of the first 8 terms.

$r^2 = \frac{81}{9} = 9$ so $r = 3$

$a r^2 = 9$ so $a = 1$

$S_8 = \frac{a(r^8 - 1)}{(r - 1)} = \frac{1(6561 - 1)}{2}$

Answer: r = 3, a = 1, S_8 = 3280

5 Conservation of momentum (CIE)

Particles A (6 kg) and B (4 kg) move in the same direction on a smooth horizontal line at 9 m s^{-1} and 8 m s^{-1}. They collide and coalesce. Find (a) the speed of the combined particle, (b) the kinetic energy lost in the collision.

Conservation of momentum (A's direction positive):
$6\times9 + 4\times8 = 10v$, so $v = 8.6$
KE before $= \frac{1}{2}\times6\times9^2 + \frac{1}{2}\times4\times8^2 = 371$
KE after $= \frac{1}{2}\times10\times8.6^2 = 369.8$
Loss $= 371 - 369.8$

Answer: (a) 8.6 m s^{-1} (b) 1.2 J

Day 347 **A Level Maths: A Tier 10-a-day: Model Solutions**

6 Collinear points and ratios (3D) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A, B and C have position vectors $\begin{pmatrix}-4\\1\\-2\end{pmatrix}$, $\begin{pmatrix}-4\\0\\-3\end{pmatrix}$ and $\begin{pmatrix}-4\\k\\-1\end{pmatrix}$ relative to O, where k is a constant. Given that A, B and C are collinear, find k and the ratio AB : BC.

AB = $\begin{pmatrix}0\\-1\\-1\end{pmatrix}$, and AC = $\begin{pmatrix}0\\k-1\\1\end{pmatrix}$

AC = λAB: component 2 gives $1 = -1\ \lambda$, so $\lambda = -1$

Hence $k = 1 + (-1) \times (-1) = 2$

AC = −1 AB, so BC = −2 AB: AB : BC = 1 : 2

Answer: k = 2, AB : BC = 1 : 2

7 Finding mean and standard deviation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim N(\mu, \sigma^2)$, with $P(X < 57) = 0.0668$ and $P(X > 64) = 0.0228$.

Find μ and σ.

z-values: 57: $z = -1.50$; 64: $z = 2.00$

$57 = \mu - 1.50\ \sigma$ and $64 = \mu + 2.00\ \sigma$

Subtract: $7 = 3.50\ \sigma$, so $\sigma = 2.00$

$\mu = 57 + 1.50 \times 2.00 = 60.0$

Answer: μ = 60, σ = 2

8 Greatest and least values using R-form (All boards)

$g(x) = \dfrac{1}{21 + 16 \sin x + 12 \cos x}$

(a) Write 16 sin x + 12 cos x in the form R sin(x + α), with α acute.

(b) Find the greatest and least values of g(x) and the smallest positive x at which the greatest value occurs (1 d.p.).

$R = \sqrt{16^2 + 12^2} = 20$, $\alpha = \tan^{-1}\dfrac{12}{16} = 36.9°$

Denominator = 21 + 20 sin(x + 36.9°), ranging from 1 to 41

Greatest g = $\frac{1}{1}$ (denominator least), least g = $\frac{1}{41}$

Least denominator when sin(x + 36.9°) = −1: x + 36.9° = 270°

x = 233.1°

Answer: R = 20, α = 36.9°; greatest $\frac{1}{1}$, least $\frac{1}{41}$; x = 233.1°

9 Tangent conditions (All boards)

The line y = kx − 1 is a tangent to the curve $y = x^2 + x + 3$.

(a) Find the possible values of k.

(b) For the larger value of k, find the coordinates of the point of contact.

Equate: $x^2 + x + 3 = kx - 1$

$x^2 + (1 - k)x + 4 = 0$

Tangent: discriminant = 0: $(1 - k)^2 = 16$

$1 - k = \pm 4$ so k = 5 or −3

k = 5: $x = \dfrac{k - 1}{2} = 2$, y = 9

Answer: (a) k = −3 or k = 5 (b) (2, 9)

10 Intersecting or skew lines (CIE (9709); Further Maths on other boards)

l_1: $\mathbf{r} = \begin{pmatrix} 3 \\ 5 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$

l_2: $\mathbf{r} = \begin{pmatrix} 7 \\ -1 \\ -4 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -3 \\ -1 \end{pmatrix}$

Determine whether l_1 and l_2 intersect. If they do, find the point of intersection.

Equate components: $3 + 0\lambda = 7 + 2\mu$; $5 + 1\lambda = -1 - 3\mu$; $-2 - 1\lambda = -4 - 1\mu$
Solving: $\lambda = 0$, $\mu = -2$, and all three equations are satisfied
Point of intersection = $(3, 5, -2)$
Answer: Intersect at $(3, 5, -2)$