Day 346 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Laws of logarithms in terms of p and q (All boards)

Given that $\log_a 2 = p$ and $\log_a 3 = q$,

express $\log_a\left(\frac{9\sqrt{2}}{4}\right)$ in terms of p and q.

$\frac{9\sqrt{2}}{4} = 2^{-\frac{3}{2}} \times 3^2$

$\log_a(2^{-\frac{3}{2}} \times 3^2) = -\frac{3}{2}\log_a 2 + 2\log_a 3$

$= -\frac{3}{2}p + 2q$

Answer: $-\frac{3}{2}p + 2q$

2 Transformations of a turning point (All boards)

The curve y = f(x) has a minimum point at (3, 4).
Find the coordinates of the corresponding point on:
(a) y = f(−x)
(b) y = 2f(x)
(c) y = f(2x)

(a) y = f(−x): (−3, 4)
(b) y = 2f(x): (3, 8)
(c) y = f(2x): $(\frac{3}{2}, 4)$

A stretch or translation moves the point in the stated direction only.

Answer: (a) (−3, 4); (b) (3, 8); (c) $(\frac{3}{2}, 4)$

3 Normal distribution probabilities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X, the length (mm), is normally distributed with mean 83 and standard deviation 2.
Find (a) P(X > 81.2) (b) P(81.2 < X < 83), each to 3 s.f.

(a) $z = \frac{81.2 - 83}{2} = -0.900$, $P(Z > -0.900) = 0.816$

(b) $z_1 = -0.900$, $z_2 = \frac{83 - 83}{2} = 0$

$P(81.2 < X < 83) = \Phi(0) - \Phi(-0.900) = 0.316$

Answer: (a) 0.816 (b) 0.316

4 Position vector and velocity (AQA, Edexcel, OCR, MEI, Eduqas)

A particle P has position vector $\mathbf{r} = (2t^3 - 2t)\mathbf{i} + (t^2 - 2t)\mathbf{j}$ metres at time t s. Find (a) the speed of P when t = 4, (b) the value of t when P is moving parallel to $\mathbf{i}$. Give answers to 3 s.f. if not exact.

$\mathbf{v} = d\mathbf{r}/dt = (6t^2 - 2)\mathbf{i} + (2t - 2)\mathbf{j}$
(a) t = 4: $\mathbf{v} = 94\mathbf{i} + 6\mathbf{j}$
Speed $= \sqrt{(94^2 + 6^2)} = \sqrt{8872}$
(b) Parallel to $\mathbf{i}$ means the $\mathbf{j}$ component is zero: $2t - 2 = 0$, so t = 1

Answer: (a) 94.2 m s^{-1} (b) t = 1

5 Scalar product, magnitude of a sum (CIE (9709); Further Maths on other boards)

|a| = 4, |b| = 3 and the angle between a and b is 150°.
Find the exact values of a·b and |a + b|.

$\mathbf{a}\cdot\mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos 150° = 4 \times 3 \times -\frac{\sqrt{3}}{2} = -6\sqrt{3}$

$|\mathbf{a} + \mathbf{b}|^2 = |\mathbf{a}|^2 + 2\mathbf{a}\cdot\mathbf{b} + |\mathbf{b}|^2 = 16 + -12\sqrt{3} + 9 = 25 - 12\sqrt{3}$

$|\mathbf{a} + \mathbf{b}| = \sqrt{25 - 12\sqrt{3}}$

Answer: $\mathbf{a}\cdot\mathbf{b} = -6\sqrt{3}$, $|\mathbf{a} + \mathbf{b}| = \sqrt{25 - 12\sqrt{3}}$

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6 Binomial expansion: terms with negative powers of x (All boards)

Find the coefficient of x^{-2} in the expansion of $(3x^2 - \frac{2}{x})^5$.

General term: ${}^5C_r\,(3x^2)^{5-r}\,(-2x^{-1})^r$
Power of x: $2(5 - r) - 1r = -2$
$10 - 3r = -2$ so $r = 4$
Coefficient = ${}^5C_4 \times 3^1 \times (-2)^4 = 240$
Answer: 240

7 Equations with sin 2x and a trig function (All boards)

Solve $\sin 2x = -\cos x$ for $0 \le x \le 2\pi$.

$\sin 2x \equiv 2\sin x \cos x$: $\cos x\,(2\sin x + 1) = 0$

$\cos x = 0$: $x = \frac{\pi}{2}, \frac{3\pi}{2}$

$\sin x = -\frac{1}{2}$: $x = \frac{7\pi}{6}, \frac{11\pi}{6}$

Do not divide by cos x, as that would lose solutions

Answer: $x = \frac{\pi}{2}, \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}$

8 Normal approximation to the Poisson (CIE)

X ~ Po(25). Use a suitable approximation to find $P(X \ge 25)$, to 3 significant figures.

$\lambda > 15$, so Y ~ N(25, 25) approximately
Continuity correction: P(Y > 24.5)
$z = -0.100$, so probability = 0.540
Answer: 0.540

9 Equation of a reciprocal curve from its graph (All boards)

The diagram shows the curve $y = \frac{k}{x - a} + b$ with asymptotes as shown. Find the values of k, a and b, and the coordinates where the curve crosses the axes.

Asymptotes $x = -1$, $y = 1$ so $a = -1$, $b = 1$

Through (1, 4): $4 = \frac{k}{2} + 1$

$k = (4 - 1)(2) = 6$
$x = 0$: $y = 7$; $y = 0$: $x = -7$
Answer: k = 6, a = −1, b = 1; (0, 7) and (−7, 0)

10 Projectile: perpendicular directions (AQA, Edexcel, OCR, MEI, Eduqas)

A ball is projected from the ground at 35 m s^{-1} at angle α above the horizontal, $\tan\alpha = \frac{12}{5}$. At time T s, while in flight, it is moving perpendicular to its initial direction. Find (a) T, (b) the height of the ball then. Take g = 9.8. Give answers to 3 s.f. if not exact.

Initial velocity $(\frac{175}{13}, \frac{420}{13})$. At time T the velocity is $(\frac{175}{13}, \frac{420}{13} - 9.8T)$.

Perpendicular: scalar product = 0: $(\frac{175}{13})^2 + \frac{420}{13}(\frac{420}{13} - 9.8T) = 0$

$35^2 - 9.8T \times \frac{420}{13} = 0$ (using $\sin^2\alpha + \cos^2\alpha = 1$)

(a) $T = 35 \div (9.8 \times \frac{12}{13}) = \frac{325}{84}$

(b) $h = \frac{420}{13} \times \frac{325}{84} - 4.9 \times \frac{325}{84}^2 = \frac{14875}{288}$

Answer: (a) T = 3.87 s (b) 51.6 m