Day 345 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Venn diagrams and conditional probability (All boards)

P(A) = 0.45, P(B) = 0.5 and P(A ∩ B) = 0.15.
Find
(a) P(A ∪ B)
(b) P(A | B′)
(c) P(A | B)

(a) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.45 + 0.5 - 0.15 = 0.8$

(b) $P(A \mid B') = \frac{P(A \cap B')}{P(B')} = \frac{0.45 - 0.15}{1 - 0.5} = 0.6$

(c) $P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{0.15}{0.5} = 0.3$

Answer: (a) 0.8 (b) 0.6 (c) 0.3

2 Perpendicular vectors and a constant (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The vectors $\begin{pmatrix} k \\ k \\ 4 \end{pmatrix}$ and $\begin{pmatrix} k \\ -6 \\ 2 \end{pmatrix}$ are perpendicular.

Find the possible values of k and, for the larger value, the exact length of the first vector.

Scalar product = 0: $k^2 - 6k + 8 = 0$
$(k - 4)(k - 2) = 0$

k = 2 or k = 4. For k = 4 the first vector is $\begin{pmatrix} 4 \\ 4 \\ 4 \end{pmatrix}$, with length $\sqrt{48} = 4\sqrt{3}$

Answer: k = 2 or 4; length $4\sqrt{3}$

3 Triangle on a coordinate grid (All boards)

The diagram shows triangle ABC.
(a) Find the equation of the line through C perpendicular to AB.
(b) Hence find the coordinates of D, the foot of the perpendicular from C to AB, and the area of triangle ABC.

Gradient AB = 0, so gradient of the perpendicular = undefined (vertical)
Through C: x = 1
Line AB: y − 1 = 0(x − −1). Solve to get D = (1, 1)
Area = ½ × AB × CD = 12

Answer: (a) x = 1 (b) D(1, 1), area 12

4 Small angle approximations: limits (AQA, Edexcel, OCR, MEI, Eduqas)

For small θ (in radians), use the small angle approximations to find the value that $\frac{\sin 4\theta \tan \theta}{1 - \cos \theta}$ approaches as $\theta \to 0$.

$\sin 4\theta \approx 4\theta$, $\tan \theta \approx \theta$, $1 - \cos \theta \approx \frac{1}{2}(\theta)^2$

Expression $\approx \frac{4\theta \times \theta}{\frac{1}{2}\theta^2} = \frac{4\theta^2}{\frac{1}{2}\theta^2} = 8$

Answer: 8

5 Work-energy on a rough slope (CIE)

A 4 kg block is released from rest on a rough plane inclined at α, $\tan\alpha = \frac{3}{4}$, $\mu = 0.25$, and slides 8 m down a line of greatest slope. Use work and energy to find (a) the work done against friction, (b) its speed after 8 m. Take $g = 10$.

$R = 4g\cos\alpha = 32$ N, $F = \mu R = 8$ N
(a) Work done against friction $= F \times d = 8 \times 8$
(b) Loss of PE $= 4g \times 8\sin\alpha = 192$ J
Work-energy: $\frac{1}{2}\times 4 \times v^2 = 192 - 64 = 128$
$v^2 = 64$
Answer: (a) 64 J (b) 8 m s^{-1}

6 Exponential growth and decay modelling (All boards)

The number of fish in a lake is modelled by $N = 500\,e^{kt}$, where t is time in years.
After 4 years, $N = 1000$.
Find (a) the exact value of k, (b) the time for N to reach 50000, to 3 s.f.

$500\,e^{4k} = 1000$, so $e^{4k} = 2$ and $k = \frac{\ln 2}{4}$

$k = 0.173$ (3 s.f.)

$500\,e^{kt} = 50000$ gives $kt = \ln 100$, so $t = \frac{\ln 100}{k} = 26.6$ years

Answer: (a) $k = \frac{\ln 2}{4}$ (b) t = 26.6 years

7 Bayes' theorem in context (All boards)

3% of people have a disease. The screening test is positive for 80% of those who have it and for 10% of those who do not.
(a) Find the probability that a person tests positive.
(b) Given a positive test, find the probability that the person has the disease (3 s.f.).

(a) $P(+) = 0.03 \times 0.8 + 0.97 \times 0.1$
$= 0.024 + 0.097 = 0.121$
(b) $P(\text{has it} \mid +) = \frac{0.03 \times 0.8}{0.121}$
$= 0.198$
Answer: (a) 0.121 (b) 0.198

8 Binomial expansion: product with a bracket (All boards)

Find the coefficient of x^4 in the expansion of
$(2 - 3x)(1 - x)^6$.

In $(1 - x)^6$: coefficient of x^3 is -20, of x^4 is 15
Coefficient $= 2 \times 15 + (-3) \times (-20)$
Answer: 90

9 Connected particles (rough table, string slack) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Particle A (3 kg) lies on a rough table, $\mu = 0.1$, joined by a light string over a smooth pulley to particle B (5 kg), 2 m above the floor. Released from rest; B lands without rebounding. Find (a) the acceleration of A before B lands, (b) the further distance A moves. Take $g = 9.8$. Give answers to 3 s.f. if not exact.

B: $5g - T = 5a$; A: $T - 0.1\times 3g = 3a$
Adding: $(5 - 0.1\times 3)g = 8a$, so $a = 5.7575$
Speed of B and A as B lands: $v^2 = 2\times 5.7575\times 2 = 23.03$
After B lands, A decelerates at $\mu g = 0.1\times 9.8 = 0.98$
$0 = v^2 - 2(\mu g)s$, $s = 23.03 \div 1.96$
Answer: (a) 5.76 m s^{-2} (b) 11.75 m

10 Foot of a perpendicular (All boards)

The line l has equation $2x - y = 7$.

(a) Find the coordinates of the foot of the perpendicular from $P(-1, -4)$ to l.

(b) Hence find the shortest distance from P to l, in exact form.

Gradient of l = 2, so perpendicular gradient = $-\frac{1}{2}$

Perpendicular through P: $-x - 2y = 9$

Solve with l: $x = 1$, $y = -5$

Distance^2 = $(-1 - 1)^2 + (-4 - (-5))^2 = 5$

Answer: (a) (1, −5) (b) $\sqrt{5}$