Day 343 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Exact definite integrals of standard functions (All boards)

Find the exact value of

$\int_1^3 \left(\frac{4}{x} + \frac{5}{x^2}\right) dx$.

Integrate: $[4 \ln x - \frac{5}{x}]$

Substitute the limits and subtract: $\frac{12 \ln 3 + 10}{3}$

Answer: $\mathbf{\frac{12 \ln 3 + 10}{3}}$

2 Stratified sampling (AQA, Edexcel, OCR, MEI, Eduqas)

A stratified sample of 80 people is taken from three groups.
9 of the sample are from Sales, which has 135 people.
(a) Find the total number of people.
(b) Admin has 165 people. How many are sampled?

Sales sampled fraction = $\frac{9}{135} = \frac{1}{15}$

Total = $80 \div \frac{1}{15} = 1200$

Admin: $165 \times \frac{1}{15} = 11$

Answer: (a) 1200 (b) 11

3 Arc length and sector area (All boards)

A sector of a circle has arc length 3.2 cm and area 6.4 cm^2.
Find the radius of the circle and the angle of the sector, in radians.

Area = $\frac{1}{2}rs$: $6.4 = \frac{1}{2} \times r \times 3.2$

$r = \frac{2 \times 6.4}{3.2} = 4$ cm

$\theta = \frac{s}{r} = \frac{3.2}{4} = 0.8$ rad

Answer: r = 4 cm, θ = 0.8 rad

4 Indices: solving equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $25^{3x-3} = 5^{3x+2}$.

Write both sides as powers of 5.
$(5^2)^{3x-3} = (5^1)^{3x+2}$
$2(3x - 3) = 3x + 2$
$6x - 6 = 3x + 2$
$3x = 8$

Answer: $\mathbf{x = \frac{8}{3}}$

5 Lift and cable tension (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A lift of mass 200 kg carries a person of mass 70 kg. The cable tension is 2808 N and the lift is accelerating. Find (a) the magnitude and direction of the acceleration, (b) the force of the floor on the person. Take g = 9.8.

Whole system (mass 270 kg), taking upwards as positive: $T - 270\,g = 270\,a$
$2808 - 2646 = 270\,a$, so $a = 0.6$
(a) a is positive, so the acceleration is 0.6 m s^{-2} upwards
(b) Person: $R - 70\,g = 70\,a$, so $R = 70(9.8 + 0.6)$

Answer: (a) 0.6 m s^{-2} upwards (b) 728 N

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6 Proving trigonometric identities (All boards)

Prove the identity

$\frac{1}{1-\sin 2\theta} + \frac{1}{1+\sin 2\theta} \equiv 2\sec^2 2\theta$

LHS $= \frac{(1+\sin 2\theta)+(1-\sin 2\theta)}{(1-\sin 2\theta)(1+\sin 2\theta)} = \frac{2}{1-\sin^2 2\theta}$

$= \frac{2}{\cos^2 2\theta} = 2\sec^2 2\theta$ = RHS

Answer: Proved

7 Three-set Venn diagram counts (All boards)

Of 41 students, 20 study Maths (M), 15 Physics (P) and 18 Chemistry (C).
9 study M and P, 8 study M and C, 7 study P and C, and 4 study all three.
One student is chosen at random. Given that a student studies exactly two subjects, find the probability that they study Physics.

Venn regions: M only 7, P only 3, C only 7,
MP only 5, MC only 4, PC only 3, all three 4, outside all 8
Exactly two subjects: 5 + 4 + 3 = 12
Of these, Physics: 5 + 3 = 8

Probability $= \frac{8}{12}$

Answer: $\frac{2}{3}$

8 Circle through two points with centre on a line (AQA, Edexcel, OCR, MEI, Eduqas)

A circle passes through A(−4, −16) and B(−4, 10).

Its centre lies on the line $y = \frac{1}{2}x - 1$.

Find the equation of the circle.

Centre $(h, \frac{1}{2}h - 1)$ is equidistant from A and B.

$(h+4)^2 + (\frac{1}{2}h + 15)^2 = (h+4)^2 + (\frac{1}{2}h - 11)^2$

Solving gives h = −4, so centre (−4, −3)

$r^2 = (0)^2 + (-13)^2 = 169$

Answer: $\mathbf{(x+4)^2 + (y+3)^2 = 169}$

9 Binomial expansion: approximating a square root (All boards)

(a) Expand $(1+2x)^{\frac{1}{2}}$ in ascending powers of x up to x^2.

(b) Use x = 0.005 to estimate $\sqrt{101}$ to 3 decimal places.

$(1+y)^{\frac{1}{2}} = 1 + \frac{1}{2}y - \frac{1}{8}y^2$, y = 2x

(a) $1 + x - \frac{1}{2}x^2$

$x = \frac{1}{200}$: $101 = 10^2(1 + \frac{1}{100})$ so $\sqrt{101} = 10 \times (1 + \frac{1}{100})^{\frac{1}{2}}$

$\approx 10 \times (1 + \frac{1}{200} - \frac{1}{80000}) = 10.049875$

Answer: (a) $\mathbf{1 + x - \frac{1}{2}x^2}$ **(b) 10.050**

10 Projectile: perpendicular directions (AQA, Edexcel, OCR, MEI, Eduqas)

A ball is projected from the ground at 38 m s^{-1} at angle α above the horizontal, $\tan\alpha = \frac{12}{5}$. At time T s, while in flight, it is moving perpendicular to its initial direction. Find (a) T, (b) the height of the ball then. Take g = 9.8. Give answers to 3 s.f. if not exact.

Initial velocity $(\frac{190}{13}, \frac{456}{13})$. At time T the velocity is $(\frac{190}{13}, \frac{456}{13} - 9.8T)$.

Perpendicular: scalar product = 0: $(\frac{190}{13})^2 + \frac{456}{13}(\frac{456}{13} - 9.8T) = 0$

$38^2 - 9.8T \times \frac{456}{13} = 0$ (using $\sin^2\alpha + \cos^2\alpha = 1$)

(a) $T = 38 \div (9.8 \times \frac{12}{13}) = \frac{1235}{294}$

(b) $h = \frac{456}{13} \times \frac{1235}{294} - 4.9 \times \frac{1235}{294}^2 = \frac{30685}{504}$

Answer: (a) T = 4.20 s (b) 60.9 m