Day 342 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Equations with sec x and tan x (All boards)

Solve $3\sec^2\theta - 18\tan\theta + 21 = 0$
for $-180° \le \theta \le 180°$. Give answers to 1 d.p.

Use $\sec^2\theta \equiv 1 + \tan^2\theta$: $3\tan^2\theta - 18\tan\theta + 24 = 0$
$\tan\theta = 2$ or $\tan\theta = 4$
$\tan\theta = 2$: $\theta = -116.6°, 63.4°$
$\tan\theta = 4$: $\theta = -104.0°, 76.0°$
Answer: $\theta = -116.6°, -104.0°, 63.4°, 76.0°$

2 Polynomial division (All boards)

Find the quotient and the remainder when $3x^4 + x^3 + 10x^2 + 4x + 8$ is divided by $(x^2 + 2)$.

Quotient (matching leading terms in turn): $3x^2 + x + 4$
$(x^2 + 2)(3x^2 + x + 4) = 3x^4 + x^3 + 10x^2 + 2x + 8$
Remainder $= 3x^4 + x^3 + 10x^2 + 4x + 8 - (3x^4 + x^3 + 10x^2 + 2x + 8) = 2x$
Answer: quotient $3x^2 + x + 4$, remainder $2x$

3 Power, driving force and resistance (CIE)

A 1500 kg car has engine power 60 kW and constant resistance 800 N. Find (a) its greatest steady speed on a horizontal road, (b) its acceleration at 20 m s^{-1} up a hill where $\sin\alpha = \frac{1}{20}$. Take g = 10. Give answers to 3 s.f. if not exact.

(a) Steady speed: driving force = resistance. $P = Fv$: $60000 = 800\,v$
(b) Driving force $F = P \div v = 60000 \div 20 = 3000$ N
Up the hill: $F - 800 - 1500\,g\sin\alpha = 1500\,a$
$3000 - 800 - 750 = 1500\,a$
$a = \frac{29}{30}$
Answer: (a) 75 m s^{-1} (b) 0.967 m s^{-2}

4 Perpendicular bisector and area (All boards)

A is (3, 6) and B is (5, −2).
(a) Find the equation of the perpendicular bisector of AB, in the form ax + by = c.
(b) The perpendicular bisector meets the x-axis at P. Find the area of triangle ABP.

Midpoint of AB = (4, 2); gradient AB = −4
Perpendicular gradient $= \frac{1}{4}$
Equation: $x - 4y = -4$
$y = 0$: P = (−4, 0)
Area $= \frac{1}{2}|(5 - 3)(0 - 6) - (-4 - 3)(-2 - 6)|$
Answer: (a) $x - 4y = -4$ (b) 34

5 Inverse trigonometric functions (AQA, Edexcel, OCR, MEI, Eduqas)

$f(x) = 3\arcsin(5x)$.
State the domain and range of f, and find $f^{-1}(x)$.

arcsin has domain −1 to 1, so $-1 \le 5x \le 1$
Range of arcsin is $-\frac{\pi}{2}$ to $\frac{\pi}{2}$, so f(x) runs from $-\frac{3\pi}{2}$ to $\frac{3\pi}{2}$
$y = 3\arcsin(5x)$ gives $\sin\frac{y}{3} = 5x$, so $f^{-1}(x) = \frac{1}{5}\sin\frac{x}{3}$
Answer: domain $-\frac{1}{5} \le x \le \frac{1}{5}$, range $-\frac{3\pi}{2} \le f(x) \le \frac{3\pi}{2}$, $f^{-1}(x) = \frac{1}{5}\sin\frac{x}{3}$

6 Binomial mean and variance (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X has a binomial distribution with mean 3 and variance 2.4.
(a) Find n and p.
(b) Find P(X = 2) to 3 significant figures.

(a) np = 3 and npq = 2.4

$q = \frac{npq}{np} = \frac{2.4}{3} = 0.8$, so p = 0.2

$n = \frac{3}{0.2} = 15$

(b) $P(X = 2) = C(15, 2)(0.2)^2(0.8)^{13} = 0.231$

Answer: (a) n = 15, p = 0.2 (b) 0.231

7 Trigonometric equations (quadratic in sin or cos) (All boards)

Solve $2\cos^2 x + 3\sin x - 3 = 0$
for $0° \leq x \leq 360°$.

Use $\cos^2 x \equiv 1 - \sin^2 x$: $2\sin^2 x - 3\sin x + 1 = 0$

Solving the quadratic: $\sin x = 1$ or $\sin x = \frac{1}{2}$

$\sin x = 1$: $x = 90°$

$\sin x = \frac{1}{2}$: $x = 30°, 150°$

Answer: x = 30°, 90°, 150°

8 Binomial in two stages (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Eggs are defective with probability 0.15, independently. A tray of 9 eggs is rejected if it has at least 2 defective. From 6 such trays, find the probability that exactly 3 are rejected (3 s.f.).

Y ~ B(9, 0.15). P(rejected) = $P(Y \geq 2) = 1 - 0.599 = 0.401$
W ~ B(6, 0.401) is the number rejected
$P(W = 3) = C(6, 3)(0.401)^3(1 - 0.401)^3 = 0.277$

Answer: 0.277

9 Projectile: perpendicular directions (AQA, Edexcel, OCR, MEI, Eduqas)

A ball is projected from the ground at 26 m s^{-1} at angle α above the horizontal, $\tan\alpha = \frac{15}{8}$. At time T s, while in flight, it is moving perpendicular to its initial direction. Find (a) T, (b) the height of the ball then. Take g = 9.8. Give answers to 3 s.f. if not exact.

Initial velocity $(\frac{208}{17}, \frac{390}{17})$. At time T the velocity is $(\frac{208}{17}, \frac{390}{17} - 9.8T)$.

Perpendicular: scalar product = 0: $(\frac{208}{17})^2 + \frac{390}{17}(\frac{390}{17} - 9.8T) = 0$

$26^2 - 9.8T \times \frac{390}{17} = 0$ (using $\sin^2\alpha + \cos^2\alpha = 1$)

(a) $T = 26 \div (9.8 \times \frac{15}{17}) = \frac{442}{147}$

(b) $h = \frac{390}{17} \times \frac{442}{147} - 4.9 \times \frac{442}{147}^2 = \frac{7774}{315}$

Answer: (a) T = 3.01 s (b) 24.7 m

10 Binomial expansion: product with a bracket (All boards)

Find the coefficient of x^3 in the expansion of
$(2 + 2x)(2 + 3x)^6$.

In $(2 + 3x)^6$: coefficient of x^2 is 2160, of x^3 is 4320
Coefficient = 2 × 4320 + 2 × 2160

Answer: 12960