Day 341 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Particle supported by two strings (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A 8 kg particle hangs in equilibrium from two light strings, as shown. Find the tension in each string (T_1 in the string at angle α). Take g = 9.8. Give answers to 3 s.f. if not exact.

$\sin\alpha = 0.8$, $\cos\alpha = 0.6$, $\sin\beta = \frac{5}{13}$, $\cos\beta = \frac{12}{13}$

Horizontal: $T_1\cos\alpha = T_2\cos\beta$, so $T_2 = 0.65\,T_1$

Vertical: $T_1\sin\alpha + T_2\sin\beta = 8\,g = 78.4$

$T_1(0.8 + 0.25) = 78.4$

$T_1 = 74.7$, $T_2 = 48.5$

Answer: T_1 = 74.7 N, T_2 = 48.5 N

2 Symmetric interval about the mean (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X, the score (marks), is $N(70, 12^2)$.

Find the symmetric limits about the mean between which the central 99% of values lie (3 s.f.).

Each tail has area 0.005, so $P(Z < z) = 0.995$

$z = 2.58$

Limits: $70 \pm 2.58 \times 12$

$= 39.1$ and 101

Answer: 39.1 to 101

3 Double and triple angle values (All boards)

θ is acute and $\tan\theta = \frac{1}{3}$.

Show that $\cos 2\theta = \frac{4}{5}$ and find the exact value of $\sin 2\theta$.

$$\cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta} = \frac{1-\frac{1}{3}^2}{1+\frac{1}{3}^2} = \frac{4}{5}$$

$$\sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta} = \frac{2\times\frac{1}{3}}{1+\frac{1}{3}^2} = \frac{3}{5}$$

Answer: $\cos 2\theta = \frac{4}{5}$, $\sin 2\theta = \frac{3}{5}$

4 Equations quadratic in disguise (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $x - 6\sqrt{x} + 8 = 0$.

Let $u = \sqrt{x}$: $u^2 - 6u + 8 = 0$

$(u - 2)(u - 4) = 0$ so u = 2 or 4

$\sqrt{x} = 2$ gives x = 4; $\sqrt{x} = 4$ gives x = 16

Answer: x = 4 or x = 16

5 Collinear points and ratios (3D) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A, B and C have position vectors $\begin{pmatrix}2\\2\\-2\end{pmatrix}$, $\begin{pmatrix}-1\\-1\\-2\end{pmatrix}$ and $\begin{pmatrix}k\\-4\\-2\end{pmatrix}$

relative to O, where k is a constant. Given that A, B and C are collinear, find k and the ratio AB : BC.

$\text{AB} = \begin{pmatrix}-3\\-3\\0\end{pmatrix}$, and $\text{AC} = \begin{pmatrix}k-2\\-6\\0\end{pmatrix}$

$\text{AC} = \lambda\text{AB}$: component 1 gives $-6 = -3\lambda$, so $\lambda = 2$

Hence $k = 2 + 2\times(-3) = -4$

AC = 2 AB, so BC = 1 AB: AB : BC = 1 : 1

Answer: k = −4, AB : BC = 1 : 1

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6 Normal approximation to the binomial (All boards)

X ~ B(150, 0.5). Use a normal approximation to estimate P(X ≥ 81) to 3 significant figures.

np = 75, npq = 37.5, so Y ~ N(75, 37.5)
With continuity correction: P(Y > 80.5)
$z = \frac{80.5 - 75}{6.12} = 0.898$, P(Z > 0.898) = 0.185
Answer: 0.185

7 Equations with cosec x and cot x (All boards)

Solve $3\text{cosec}^2 x + 9\cot x - 15 = 0$
for $-180° < x < 180°$, $x \neq 0$. Give answers to 1 d.p.

Use $\text{cosec}^2 x \equiv 1 + \cot^2 x$: $3\cot^2 x + 9\cot x - 12 = 0$
cot x = 1 or cot x = −4
cot x = 1 means tan x = 1: x = −135.0°, 45.0°
cot x = −4 means $\tan x = -\frac{1}{4}$: x = −14.0°, 166.0°
Answer: x = −135.0°, −14.0°, 45.0°, 166.0°

8 Tangent conditions (All boards)

Find the set of values of k for which the line y = 4x + k does not meet the curve $y = 2x^2 + 3x + 3$.

$2x^2 + 3x + 3 = 4x + k$ gives $2x^2 - x + (3 - k) = 0$
Discriminant: $(-1)^2 - 4 \times 2(3 - k) = -23 + 8k$
No intersection: discriminant < 0
Answer: $k < \frac{23}{8}$

9 Ladder in limiting equilibrium (AQA, Edexcel, OCR, MEI)

A uniform ladder AB (5 m, 10 kg) rests with A on rough ground, μ = 0.2, and B against a smooth wall, B being 3 m above the ground. A man of mass 70 kg climbs the ladder. How far along the ladder from A can he climb before it slips?

Foot of ladder is $\sqrt{(5^2 - 3^2)} = 4$ m from the wall
Vertical: R = (10 + 70)g. Limiting friction: F = μR = 0.2(10 + 70)g
Horizontal: S = F (wall reaction S)
Moments about A (man at distance x along ladder, horizontal distance x×4/5):
S×3 = 10 g×2 + 70 g×(4 x ÷ 5)
16×3 = 20 + 56 x (g cancels), so x = 0.5
Answer: 0.5 m

10 Circle and line: chord length (AQA, Edexcel, OCR, MEI, Eduqas)

The line 4x + 3y = 3 meets the circle $(x - 4)^2 + (y - 2)^2 = 49$ at the points A and B. Find the exact length of AB.

Centre (4, 2), radius 7. Distance from centre to line:
$d = \frac{|19|}{5} = \frac{19}{5}$

$\text{Half-chord}^2 = 49 - (\frac{19}{5})^2 = \frac{864}{25}$

$AB = 2\sqrt{\frac{864}{25}}$

Answer: $\frac{120\sqrt{6}}{25}$