Day 34 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Factor theorem: roots with surds (All boards)

$f(x) = 3x^3 - 6x^2 - 33x - 18$.
(a) Show that $(x + 2)$ is a factor of $f(x)$.
(b) Solve $f(x) = 0$, giving surds in simplest form.

$f(-2) = -24 - 24 + 66 - 18 = 0$, so $(x + 2)$ is a factor.

$f(x) = 3(x + 2)(x^2 - 4x - 3)$

$x^2 - 4x - 3 = 0$ gives $x = \frac{4 \pm \sqrt{28}}{2}$

$\sqrt{28} = 2\sqrt{7}$

Answer: $x = -2$ or $x = 2 \pm \sqrt{7}$

2 Fixed point iteration (All boards)

The equation $e^x = 2x + 5$ has a root α. Use the iteration $x_{n+1} = \ln(2x + 5)$ with $x_0 = 1$ to find x_1, x_2 and x_3 to 4 d.p., and hence the value of α to 2 d.p. (continue until the 2 d.p. value is settled).

$x_1 = \ln(2 \times 1 + 5) = 1.9459$

$x_2 = 2.1851$, $x_3 = 2.2375$

Continuing, the values settle at 2.2516, so $\alpha = 2.25$

Answer: $x_1 = 1.9459$, $x_2 = 2.1851$, $x_3 = 2.2375$; $\alpha = 2.25$

3 Estimating mean and standard deviation (All boards)

The table summarises the time, t seconds, to complete a task.
Estimate the mean and standard deviation (3 s.f.).

Midpoints: 10, 30, 50, 70, 90, 110 ($n = 45$)

$\Sigma fx = 3430$, so mean $= \frac{3430}{45} = 76.2$

$\Sigma fx^2 = 304500$

$\text{sd} = \sqrt{\frac{304500}{45} - (76.2)^2} = 30.9$

Answer: mean 76.2, sd 30.9

4 Newton's second law with vectors (AQA, Edexcel, OCR, MEI, Eduqas)

A particle of mass 4 kg is acted on by forces $\mathbf{F_1} = (-7\mathbf{i} - 28\mathbf{j})$ N and $\mathbf{F_2}$. Its acceleration is $(-4\mathbf{i} - 4\mathbf{j})$ m s^{-2}. Find (a) $\mathbf{F_2}$, (b) the angle between $\mathbf{F_2}$ and $\mathbf{i}$, to 1 d.p.

$\mathbf{F} = m\mathbf{a}$: $\mathbf{F_1} + \mathbf{F_2} = 4(-4\mathbf{i} - 4\mathbf{j}) = -16\mathbf{i} - 16\mathbf{j}$

(a) $\mathbf{F_2} = (-16\mathbf{i} - 16\mathbf{j}) - (-7\mathbf{i} - 28\mathbf{j}) = -9\mathbf{i} + 12\mathbf{j}$

(b) $|\mathbf{F_2}| = \sqrt{(-9^2 + 12^2)} = 15$

$\cos\theta = -9 \div 15$, so $\theta = 126.9°$

Answer: (a) $\mathbf{F_2} = (-9\mathbf{i} + 12\mathbf{j})$ N (b) 126.9°

5 Change of sign to locate a root (All boards)

$f(x) = x^3 - 4x - 11$.
Show that the equation $f(x) = 0$ has a root α in the interval $[2, 3]$.

$f(x)$ is continuous on the interval
$f(2) = -11.0 < 0$
$f(3) = 4.00 > 0$
A change of sign, so a root lies in $[2, 3]$

Answer: Sign change: $f(2) = -11.0$, $f(3) = 4.00$

6 Circle touching both axes (AQA, Edexcel, OCR, MEI, Eduqas)

A circle touches both coordinate axes and passes through the point (2, −1). Its centre lies in the fourth quadrant. Find the equations of the two possible circles.

Centre (r, −r), radius r > 0.

Through (2, −1): $(2 - r)^2 + (-1 + r)^2 = r^2$

$r^2 - 6r + 5 = 0$

$(r - 1)(r - 5) = 0$, so r = 1 or 5

Answer: $(x - 1)^2 + (y + 1)^2 = 1$ or $(x - 5)^2 + (y + 5)^2 = 25$

7 Conditional normal probability (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim N(66, 5^2)$. Find $P(X > 76 \mid X > 70)$ to 3 significant figures.

Since X > 76 implies X > 70: $P = \frac{P(X > 76)}{P(X > 70)}$

P(X > 76) = P(Z > 2.00) = 0.0228

P(X > 70) = P(Z > 0.800) = 0.212

$P = \frac{0.0228}{0.212} = 0.107$

Answer: 0.107

8 Friction: find μ then stopping distance (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A 4 kg box is pulled from rest along rough horizontal ground by a horizontal force of 11.04 N and moves 10 m in 5 s. Find (a) the coefficient of friction. The force is then removed; (b) find the further distance moved. Take g = 9.8. Give answers to 3 s.f. if not exact.

$s = \frac{1}{2}at^2$: $10 = \frac{1}{2}a \times 5^2$, so a = 0.8

(a) P − μ4 g = 4 a: 11.04 − 39.2 μ = 3.2, so μ = 0.2

(b) v = at = 0.8×5 = 4; after removal deceleration = μg = 1.96

$0 = 4^2 - 2 \times 1.96 \times s$, s = 4.08

Answer: (a) μ = 0.2 (b) 4.08 m

9 Surds: equation with surd coefficients (AQA, Edexcel, OCR, MEI, Eduqas)

Solve $x(\sqrt{7} - 3) = 1 - 3\sqrt{7}$, writing x in the form $p + q\sqrt{7}$.

$x = \frac{1 - 3\sqrt{7}}{\sqrt{7} - 3}$

Multiply top and bottom by $\sqrt{7} + 3$.

Numerator: $(1 - 3\sqrt{7})(\sqrt{7} + 3) = -18 - 8\sqrt{7}$

Denominator: 7 − 9 = −2

Answer: $x = 9 + 4\sqrt{7}$

10 Intersecting or skew lines (CIE (9709); Further Maths on other boards)

l_1: $\mathbf{r} = \begin{pmatrix} 2 \\ 7 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -3 \\ 3 \end{pmatrix}$

l_2: $\mathbf{r} = \begin{pmatrix} 3 \\ 4 \\ 8 \end{pmatrix} + \mu \begin{pmatrix} -2 \\ -2 \\ -2 \end{pmatrix}$

Determine whether l_1 and l_2 intersect. If they do, find the point of intersection.

Equate components: $2 + 1\lambda = 3 - 2\mu$; $7 - 3\lambda = 4 - 2\mu$; $3 + 3\lambda = 8 - 2\mu$

Solving two of the equations gives a unique (λ, μ)

which does not satisfy the third, so the lines do not meet

Answer: The lines do not intersect (skew)