Day 339 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Trapezium rule with over or under estimate (AQA, Edexcel, OCR, MEI, Eduqas)

Use the trapezium rule with 4 strips to estimate $\int_0^2 2(2^x)\,dx$, giving your answer to 3 s.f.

State, with a reason, whether your estimate is an overestimate or an underestimate.

h = $\frac{1}{2}$; y-values at x = 0, $\frac{1}{2}$, 1, $\frac{3}{2}$, 2: 2.0000, 2.8284, 4.0000, 5.6569, 8.0000

T = $\frac{1}{4}$[2.0000 + 8.0000 + 2(2.8284 + 4.0000 + 5.6569)]

T = 8.74

The curve is convex (y'' > 0) on this interval, so the chords lie above the curve: overestimate

Answer: 8.74, an overestimate

2 Venn diagrams and conditional probability (All boards)

P(A) = 0.3, P(B) = 0.6 and P(A ∩ B) = 0.1.

Find

(a) P(A ∪ B)

(b) P(B′ | A′)

(c) P(B | A)

(a) P(A ∪ B) = P(A) + P(B) − P(A ∩ B) = 0.3 + 0.6 − 0.1 = 0.8

(b) P(B′ | A′) = $\frac{P(A' \cap B')}{P(A')} = \frac{1 - 0.8}{1 - 0.3} = \frac{2}{7}$

(c) P(B | A) = $\frac{P(A \cap B)}{P(A)} = \frac{0.1}{0.3} = \frac{1}{3}$

Answer: (a) 0.8 (b) $\frac{2}{7}$ (c) $\frac{1}{3}$

3 Completing the square (All boards)

f(x) = x^2 − 10x + 20.

(a) Write f(x) in the form $a(x + p)^2 + q$.

(b) State the minimum value of f(x) and the value of x at which it occurs.

(c) Solve f(x) = 0, giving surds in simplest form.

f(x) = $(x - 5)^2$ − 5

(b) minimum value −5 when x = 5

(c) $(x - 5)^2 = 5$, so $(x - 5)^2 = 5$

$x - 5 = \pm\sqrt{5}$

Answer: (a) $(x - 5)^2 - 5$ (b) minimum −5 at x = 5 (c) $x = 5 \pm \sqrt{5}$

4 Newton-Raphson method (AQA, Edexcel, OCR, MEI, Eduqas)

f(x) = e^x + 2x − 8.

Use the Newton-Raphson method with x_0 = 1.5 to find x_1 and x_2, giving your answers to 4 d.p.

f′(x) = e^x + 2

$x_1 = 1.5 - \frac{f(1.5)}{f'(1.5)} = 1.5 - \frac{-0.5183}{6.4817} = 1.5800$

$x_2 = 1.5800 - \frac{f(1.5800)}{f'(1.5800)} = 1.5778$

Answer: x_1 = 1.5800, x_2 = 1.5778

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5 Binomial expansion: terms with negative powers of x (All boards)

Find the coefficient of x^{12} in the expansion of

$(2x^2 - \frac{1}{x^2})^8$.

General term: ${}^8C_r\,(2x^2)^{8-r}\,(-x^{-2})^r$

Power of x: $2(8 - r) - 2r = 12$

$16 - 4r = 12$ so $r = 1$

Coefficient = ${}^8C_1 \times 2^7 \times (-1)^1 = -1024$

Answer: -1024

6 Pulled at an angle on rough ground (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A 4 kg box is pulled along rough horizontal ground, $\mu = \frac{1}{3}$, by a rope at angle θ above the horizontal, where $\tan\theta = \frac{5}{12}$.

The tension is 35 N. Find (a) the normal reaction, (b) the acceleration of the box. Take g = 9.8. Give answers to 3 s.f. if not exact.

$\sin\theta = \frac{5}{13}$, $\cos\theta = \frac{12}{13}$

(a) Vertical: $R + 35\sin\theta = 4\,g$, so $R = \frac{1673}{65}$

(b) Horizontal: $35\cos\theta - \mu R = 4\,a$

$\frac{420}{13} - \frac{1}{3}\times\frac{1673}{65} = 4\,a$

$a = \frac{4627}{780}$

Answer: (a) 25.7 N (b) 5.93 m s^{-2}

7 Separable differential equations (All boards)

Solve the differential equation

$(1 + x^2)\frac{dy}{dx} = xy$,

given that y = 1 when x = 0 (y > 0). Give y in terms of x.

Separate: $\int\frac{1}{y}\,dy = \int\frac{x}{1+x^2}\,dx$, so $\ln y = \frac{1}{2}\ln(1 + x^2) + C$

x = 0, y = 1 gives $C = \ln 1$

$y = \sqrt{1 + x^2}$

Answer: $y = \sqrt{1 + x^2}$

8 Recurrence relations: limits (All boards)

$u_1 = 0$ and $u_{n+1} = \frac{1}{2}u_n + 3$.

(a) Find the limit L of the sequence as $n \to \infty$.
(b) Find the least n for which $|u_n - L| < 0.01$.

(a) $L = \frac{1}{2}L + 3$ gives $L = \frac{3}{1 - \frac{1}{2}} = 6$

(b) $u_n - L = (u_1 - L)\,r^{n-1} = (-6) \times \frac{1}{2}^{n-1}$

$|-6| \times \frac{1}{2}^{n-1} < \frac{1}{100}$

Check: n = 10 gives 0.01172, n = 11 gives 0.00586

Answer: (a) L = 6 (b) n = 11

9 Two-tailed binomial test critical region (AQA, Edexcel, OCR, MEI, Eduqas)

$X \sim B(31, p)$. H_0: $p = 0.3$ is tested against H_1: $p \neq 0.3$ at the 5% level.

(a) Find the critical region.

(b) Find the actual significance level of the test (3 s.f.).

(a) Each tail ≤ 0.025. $P(X \leq 4) = 0.0239$, $P(X \leq 5) = 0.0627$

$P(X \geq 15) = 0.0239$, $P(X \geq 14) = 0.0534$

Critical region: $X \leq 4$ or $X \geq 15$

(b) Actual level = 0.0239 + 0.0239 = 0.0478

Answer: (a) $X \leq 4$ or $X \geq 15$ (b) 0.0478

10 Ladder in limiting equilibrium (AQA, Edexcel, OCR, MEI)

A uniform ladder AB (20 m, 30 kg) rests with A on rough ground, $\mu = 0.5$, and B against a smooth wall, B being 16 m above the ground. A man of mass 60 kg climbs the ladder. How far along the ladder from A can he climb before it slips?

Foot of ladder is $\sqrt{(20^2 - 16^2)} = 12$ m from the wall

Vertical: $R = (30 + 60)g$. Limiting friction: $F = \mu R = 0.5(30 + 60)g$

Horizontal: $S = F$ (wall reaction S)

Moments about A (man at distance x along ladder, horizontal distance $x \times 12/20$):

$S \times 16 = 30g \times 6 + 60g \times (12x \div 20)$

$45 \times 16 = 180 + 36x$ (g cancels), so $x = 15$

Answer: 15 m