Day 338 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Constant acceleration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A cyclist passes P with speed 4 m s^{-1} and accelerates uniformly. She travels 140 m in the next 20 s, reaching Q. Find (a) her acceleration, (b) her speed at Q, (c) her speed at the midpoint of PQ. Give answers to 3 s.f. if not exact.

(a) $s = ut + \frac{1}{2}at^2$: $140 = 4\times20 + \frac{1}{2}\times a\times20^2$, so $a = 0.3$
(b) $v = u + at = 4 + 0.3\times20 = 10$
(c) For the midpoint, $s = 70$: $v^2 = u^2 + 2as = 4^2 + 2\times0.3\times70$
$v^2 = 58$
Answer: (a) 0.3 m s^{-2} (b) 10 m s^{-1} (c) 7.62 m s^{-1}

2 Venn diagrams and conditional probability (All boards)

P(A) = 0.7, P(B) = 0.6 and P(A ∩ B) = 0.35.
Find
(a) P(A ∪ B)
(b) P(B | A)
(c) P(B′ | A′)

(a) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.7 + 0.6 - 0.35 = 0.95$
(b) $P(B \mid A) = \frac{P(A \cap B)}{P(A)} = \frac{0.35}{0.7} = 0.5$
(c) $P(B' \mid A') = \frac{P(A' \cap B')}{P(A')} = \frac{1 - 0.95}{1 - 0.7} = \frac{1}{6}$
Answer: (a) 0.95 (b) 0.5 (c) $\frac{1}{6}$

3 Index equations solved with logarithms (All boards)

Solve $2^{3x+3} = 3^{3x-3}$.
Give your answer to 3 significant figures.

Take logs: $(3x + 3)\ln 2 = (3x - 3)\ln 3$
Collect x terms: $x(3\ln 2 - 3\ln 3) = -3\ln 3 - 3\ln 2$
$x = \frac{-3\ln 3 - 3\ln 2}{3\ln 2 - 3\ln 3}$
$x = 4.42$
Answer: x = 4.42

4 Line and curve intersection (diagram) (All boards)

The diagram shows the curve $y = x^2 - 4x - 5$ and the line $y = -2x + 3$, meeting at A and B.
(a) Find the coordinates of A and B.
(b) Find the coordinates of the midpoint of AB.

$x^2 - 4x - 5 = -2x + 3$: $x^2 - 2x - 8 = 0$
$(x + 2)(x - 4) = 0$, $x = -2$ or 4
A(−2, 7), B(4, −5)
Midpoint: average the coordinates
Answer: (a) A(−2, 7), B(4, −5) (b) (1, 1)

5 Newton-Raphson method (AQA, Edexcel, OCR, MEI, Eduqas)

$f(x) = x^3 + 2x - 2$.
Use the Newton-Raphson method with $x_0 = 1$ to find x_1 and x_2, giving your answers to 4 d.p.

$f'(x) = 3x^2 + 2$
$x_1 = 1 - \frac{f(1)}{f'(1)} = 1 - \frac{1.0000}{5.0000} = 0.8000$
$x_2 = 0.8000 - \frac{f(0.8000)}{f'(0.8000)} = 0.7714$
Answer: x_1 = 0.8000, x_2 = 0.7714

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6 Poisson over different intervals (CIE)

Buses arrive at a stop at random at an average of 6 per hour.
Find the probability that in a 45-minute period there are (a) zero events (b) at least 2.
(c) Find the probability that at least one occurs in each of two separate 45-minute periods.

$\lambda = 6 \times \frac{45}{60} = 4.5$, $X \sim Po(4.5)$

(a) $P(X = 0) = e^{-4.5} = 0.0111$
(b) $P(X \geq 2) = 1 - P(X \leq 1) = 0.939$
(c) $(1 - 0.0111)^2 = 0.978$
Answer: (a) 0.0111 (b) 0.939 (c) 0.978

7 Circle through two points with centre on a line (AQA, Edexcel, OCR, MEI, Eduqas)

A circle passes through A(11, 9) and B(11, −3).
Its centre lies on the line $y = 2x - 3$.
Find the equation of the circle.

Centre $(h, 2h - 3)$ is equidistant from A and B.
$(h - 11)^2 + (2h - 12)^2 = (h - 11)^2 + (2h)^2$
Solving gives $h = 3$, so centre (3, 3)
$r^2 = (8)^2 + (6)^2 = 100$
Answer: $(x - 3)^2 + (y - 3)^2 = 100$

8 Constant acceleration in vector form (AQA, Edexcel, OCR, MEI, Eduqas)

A particle P has constant acceleration $(-2\mathbf{i} - 3\mathbf{j})$ m s^{-2}. When t = 0, P is at $(2\mathbf{i} - 4\mathbf{j})$ m moving with velocity $(4\mathbf{i} - \mathbf{j})$ m s^{-1}. Find (a) when P moves parallel to $\mathbf{j}$, (b) its position vector then.

$\mathbf{v} = \mathbf{u} + \mathbf{at} = (4\mathbf{i} - \mathbf{j}) + t(-2\mathbf{i} - 3\mathbf{j})$
(a) Parallel to $\mathbf{j}$: the $\mathbf{i}$ component is zero: $4 + -2t = 0$, so $t = 2$
(b) $\mathbf{r} = \mathbf{r}_0 + \mathbf{ut} + \frac{1}{2}\mathbf{at}^2$
$= (2\mathbf{i} - 4\mathbf{j}) + 2(4\mathbf{i} - \mathbf{j}) + 2(-2\mathbf{i} - 3\mathbf{j})$
$= 6\mathbf{i} - 12\mathbf{j}$
Answer: (a) $t = 2$ (b) $\mathbf{r} = (6\mathbf{i} - 12\mathbf{j})$ m

9 Proof by contradiction (AQA, Edexcel, OCR, MEI, Eduqas)

Prove by contradiction that there are no integers m and n such that $6m + 15n = 97$.

Assume $6m + 15n = 97$ for integers m, n.
Then $3(2m + 5n) = 97$, so 3 divides 97.
But 97 is not a multiple of 3. Contradiction.
Answer: no such integers exist

10 Greatest and least values using R-form (All boards)

$g(x) = \frac{1}{15 + 12 \sin x + 5 \cos x}$

(a) Write $12 \sin x + 5 \cos x$ in the form $R \sin(x + \alpha)$, with α acute.
(b) Find the greatest and least values of g(x) and the smallest positive x at which the greatest value occurs (1 d.p.).

$R = \sqrt{12^2 + 5^2} = 13$, $\alpha = \tan^{-1}\frac{5}{12} = 22.6°$

Denominator $= 15 + 13 \sin(x + 22.6°)$, ranging from 2 to 28

Greatest $g = \frac{1}{2}$ (denominator least), least $g = \frac{1}{28}$

Least denominator when $\sin(x + 22.6°) = -1$: $x + 22.6° = 270°$
$x = 247.4°$

Answer: $R = 13$, $\alpha = 22.6°$; greatest $\frac{1}{2}$, least $\frac{1}{28}$; $x = 247.4°$