Day 337 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Implicit differentiation: horizontal tangents (All boards)

A curve has equation $x^2 - xy + 3y^2 = 99$.
Find the coordinates of the points where the tangent is parallel to the x-axis.

$\frac{dy}{dx} = -\frac{2x - y}{-x + 6y}$

Horizontal tangent: $2x - y = 0$, so $y = 2x$

Substitute: $11x^2 = 99$, so $x^2 = 9$ and $x = \pm 3$

Points: (3, 6) and (−3, −6)

Answer: (3, 6) and (−3, −6)

2 Newton's second law with vectors (AQA, Edexcel, OCR, MEI, Eduqas)

A particle of mass 5 kg is acted on by forces $F_1 = (2\mathbf{i} - 19\mathbf{j})$ N and F_2. Its acceleration is $(-2\mathbf{i} - 2\mathbf{j})$ m s^{-2}. Find (a) F_2, (b) the angle between F_2 and $\mathbf{i}$, to 1 d.p.

$F = ma$: $F_1 + F_2 = 5(-2\mathbf{i} - 2\mathbf{j}) = -10\mathbf{i} - 10\mathbf{j}$

(a) $F_2 = (-10\mathbf{i} - 10\mathbf{j}) - (2\mathbf{i} - 19\mathbf{j}) = -12\mathbf{i} + 9\mathbf{j}$

(b) $|F_2| = \sqrt{(-12^2 + 9^2)} = 15$

$\cos\theta = -12 \div 15$, so $\theta = 143.1°$

Answer: (a) $F_2 = (-12\mathbf{i} + 9\mathbf{j})$ N (b) 143.1°

3 Double angle equations (All boards)

Solve $\cos 2x + 3\sin x = 1$
for $-180° \leq x \leq 180°$.

$\cos 2x \equiv 1 - 2\sin^2 x$: $2\sin^2 x - 3\sin x = 0$

Solve the quadratic: $\sin x = 0$ or $\sin x = \frac{3}{2}$

$\sin x = 0$: $x = -180°, 0°, 180°$

$\sin x = \frac{3}{2}$ is impossible

Answer: $x = -180°, 0°, 180°$

4 Systematic sampling (AQA, Edexcel, OCR, MEI, Eduqas)

A systematic sample is taken from a numbered list of 330 items.
The 4th item chosen is number 35 and the 6th item chosen is number 55.
Find the sampling interval, the number of the first item chosen and the size of the sample.

Interval = $(55 - 35) \div (6 - 4) = 10$

First item = $35 - 3 \times 10 = 5$

Sample size = $330 \div 10 = 33$

Answer: Interval 10, first 5, size 33

5 Power, driving force and resistance (CIE)

A 1200 kg car has engine power 40 kW and constant resistance 500 N. Find (a) its greatest steady speed on a horizontal road, (b) its acceleration at 24 m s^{-1} up a hill where $\sin\alpha = \frac{1}{20}$. Take $g = 10$. Give answers to 3 s.f. if not exact.

(a) Steady speed: driving force = resistance. $P = Fv$: $40000 = 500\,v$

(b) Driving force $F = P \div v = 40000 \div 24 = \frac{5000}{3}$ N

Up the hill: $F - 500 - 1200\,g\sin\alpha = 1200\,a$

$\frac{5000}{3} - 500 - 600 = 1200\,a$

$a = \frac{17}{36}$

Answer: (a) 80 m s^{-1} (b) 0.472 m s^{-2}

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6 Sigma notation (All boards)

Find the value of $\sum_{r=1}^{5} 5 \times 3^r$.

Geometric series: first term 15, ratio 3, 5 terms

$S = \frac{15(3^5 - 1)}{(3 - 1)}$

$= \frac{15 \times 242}{2}$

Answer: 1815

7 Conditional binomial probability (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X ~ B(14, 0.2). Find P(X = 4 | X ≥ 2) to 3 significant figures.

$P(X = 4 \mid X \geq 2) = \frac{P(X = 4)}{P(X \geq 2)}$ (since X = 4 implies X ≥ 2)

P(X = 4) = 0.172

P(X ≥ 2) = 1 − P(X ≤ 1) = 0.802

Ratio = 0.214

Answer: 0.214

8 Factor theorem and remainder theorem (All boards)

$f(x) = 2x^3 + ax^2 + bx + 48$, where a and b are constants.

(x − 3) is a factor of f(x). The remainder when f(x) is divided by (x + 1) is 40.

(a) Find a and b. (b) Solve f(x) = 0.

f(3) = 0: 54 + 9a + 3b + 48 = 0

f(−1) = 40: −2 + a − b + 48 = 40

Solving simultaneously: a = −10, b = −4

$f(x) = 2x^3 - 10x^2 - 4x + 48 = 2(x - 3)(x + 2)(x - 4)$

Answer: (a) a = −10, b = −4 (b) x = −2, 3, 4

9 Collinear points and ratios (3D) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A, B and C have position vectors $\begin{pmatrix} 2 \\ -4 \\ -4 \end{pmatrix}$, $\begin{pmatrix} 3 \\ -7 \\ -7 \end{pmatrix}$ and $\begin{pmatrix} 5 \\ -13 \\ k \end{pmatrix}$

relative to O, where k is a constant. Given that A, B and C are collinear, find k and the ratio AB : BC.

$AB = \begin{pmatrix} 1 \\ -3 \\ -3 \end{pmatrix}$, and $AC = \begin{pmatrix} 3 \\ -9 \\ k + 4 \end{pmatrix}$

AC = λAB: component 3 gives −9 = −3 λ, so λ = 3

Hence k = −4 + 3 × (−3) = −13

AC = 3 AB, so BC = 2 AB: AB : BC = 1 : 2

Answer: k = −13, AB : BC = 1 : 2

10 Partial fractions and binomial expansion (All boards)

$f(x) = \frac{16x - 6}{(1 - 3x)(1 - 2x)}$.

(a) Express f(x) in partial fractions.

(b) Hence expand f(x) up to x^2.

(c) State the range of x for which the expansion is valid.

(a) $f(x) = \frac{A}{(1 - 3x)} + \frac{B}{(1 - 2x)}$: A = −2, B = −4

(b) $-2(1 - 3x)^{-1} = -2(1 + 3x + 9x^2 \ldots)$

Add the series for both fractions.

(c) Need |3x| < 1 and |2x| < 1

Answer: (a) $\frac{-2}{(1 - 3x)} + \frac{-4}{(1 - 2x)}$ **(b)** $-6 - 14x - 34x^2$ **(c)** $|x| < \frac{1}{3}$