Day 336 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Scalar product, magnitude of a sum (CIE (9709); Further Maths on other boards)

$|a| = 7$, $|b| = 5$ and the angle between a and b is 30°.
Find the exact values of $a \cdot b$ and $|a + b|$.

$a \cdot b = |a||b|\cos 30° = 7 \times 5 \times \frac{\sqrt{3}}{2} = \frac{35\sqrt{3}}{2}$

$|a + b|^2 = |a|^2 + 2a \cdot b + |b|^2 = 49 + 35\sqrt{3} + 25 = 35\sqrt{3} + 74$

$|a + b| = \sqrt{35\sqrt{3} + 74}$

Answer: $a \cdot b = \frac{35\sqrt{3}}{2}$, $|a + b| = \sqrt{35\sqrt{3} + 74}$

2 Independent and mutually exclusive events (All boards)

P(A) = 0.1 and P(A ∪ B) = 0.55.
(a) Given that A and B are independent, find P(B).
(b) Given instead that A and B are mutually exclusive, find P(B).
(c) In the case in (a), find P(A′ ∩ B).

(a) $P(A \cup B) = P(A) + P(B) - P(A)P(B)$, so $0.55 = 0.1 + P(B)(1 - 0.1)$

$P(B) = \frac{0.55 - 0.1}{0.9} = 0.5$

(b) $P(A \cap B) = 0$, so $P(B) = 0.55 - 0.1 = 0.45$
(c) $P(A' \cap B) = P(A')P(B) = 0.9 \times 0.5 = 0.45$
Answer: (a) 0.5 (b) 0.45 (c) 0.45

3 Binomial expansion: a coefficient (All boards)

Find the coefficient of x^3 in the expansion of $(1 + 2x)^9$.

Term in x^3: ${}^9C_3 \times 1^6 \times (2x)^3$
$= 84 \times 1 \times 8$
Answer: 672

4 Collinear points and ratios (3D) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A, B and C have position vectors $\begin{pmatrix} -3 \\ -1 \\ 3 \end{pmatrix}$, $\begin{pmatrix} -4 \\ -2 \\ 6 \end{pmatrix}$ and $\begin{pmatrix} -6 \\ k \\ 12 \end{pmatrix}$

relative to O, where k is a constant. Given that A, B and C are collinear, find k and the ratio AB : BC.

$AB = \begin{pmatrix} -1 \\ -1 \\ 3 \end{pmatrix}$, and $AC = \begin{pmatrix} -3 \\ k + 1 \\ 9 \end{pmatrix}$

$AC = \lambda AB$: component 2 gives $-3 = -1\,\lambda$, so $\lambda = 3$
Hence $k = -1 + 3 \times (-1) = -4$
AC = 3 AB, so BC = 2 AB: AB : BC = 1 : 2
Answer: k = −4, AB : BC = 1 : 2

5 Connected particles on a rough incline (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The system is released from rest and A moves up the rough plane. Find (a) the acceleration, (b) the tension in the light string. Take g = 9.8. Give answers to 3 s.f. if not exact.

$\sin\alpha = \frac{12}{13}$, $\cos\alpha = \frac{5}{13}$; friction $F = \mu R = 0.4 \times 4\,g\cos\alpha$

B: $9g - T = 9a$
A: $T - 4g\sin\alpha - F = 4a$

Adding: $g(9 - \frac{48}{13} - \frac{8}{13}) = 13a$

(a) $a = 3.54$; (b) $T = 9(g - a)$
Answer: (a) 3.54 m s^{-2} (b) 56.4 N

6 Circle touching an axis (diagram) (AQA, Edexcel, OCR, MEI, Eduqas)

The diagram shows a circle that touches the x-axis at A(3, 0) and passes through B(4, 1).
Find the equation of the circle.

Touches the x-axis at A, so the centre is (3, r) and the radius is r.

B on the circle: $(1)^2 + (1 - r)^2 = r^2$

$1 + 1 - 2r = 0$ so $r = 1$

Answer: $(x - 3)^2 + (y - 1)^2 = 1$

7 Beam about to tilt (AQA, Edexcel, OCR, MEI)

A uniform plank AB (mass 15 kg, length 4 m) rests horizontally on supports at C and D, where AC = 1.5 m and DB = 0.5 m. A 5 kg particle is placed at E, AE = x m, and the plank is about to tilt about C. Find x.

About to tilt about C, so the reaction at D is zero and E lies between A and C.

Moments about C: $5\,g \times (1.5 - x) = 15\,g \times (2 - 1.5)$

$5(1.5 - x) = 7.5$

$x = 1.5 - 1.5$

Answer: $x = 0$

8 Implicit differentiation: horizontal tangents (All boards)

A curve has equation $3x^2 - xy + 2y^2 = 69$.
Find the coordinates of the points where the tangent is parallel to the x-axis.

$$\frac{dy}{dx} = -\frac{6x - y}{-x + 4y}$$

Horizontal tangent: $6x - y = 0$, so $y = 6x$

Substitute: $69x^2 = 69$, so $x^2 = 1$ and $x = \pm 1$

Points: (1, 6) and (−1, −6)

Answer: (1, 6) and (−1, −6)

9 Binomial expansion: (a + bx)^n for rational n (All boards)

Expand $(2 - x)^{-2}$ in ascending powers of x, up to and including x^2.
State the range of x for which the expansion is valid.

$$(2 - x)^{-2} = 2^{-2}\left(1 + \frac{-1x}{2}\right)^{-2}$$

$$2^{-2} = \frac{1}{4}$$

Expand $(1 + y)^{-2}$ with $y = \frac{-1x}{2}$

Valid for $\left|\frac{-1x}{2}\right| < 1$, i.e. $|x| < 2$

Answer: $\frac{1}{4} + \frac{1}{4}x + \frac{3}{16}x^2$; $|x| < 2$

10 Hypothesis test for a Poisson mean (CIE)

Faults occur as a Poisson process at a mean rate of 4 per roll of cloth.
After a change, the number recorded in one roll of cloth is 1.
Test at the 5% level whether the mean rate has decreased.

$H_0: \lambda = 4$, $H_1: \lambda < 4$; $X \sim Po(4)$

$P(X \leq 1) = 0.0916$

$0.0916 > 0.05$

Do not reject H_0: insufficient evidence of a change.

Answer: p-value 0.0916; do not reject H_0