Day 335 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Acceleration function of time (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves on the x-axis. At time t s its acceleration is $(12t - 36)$ m s^{-2}. When t = 0 it is at O with velocity 7 m s^{-1}. Find (a) its velocity when the acceleration is zero, (b) its displacement from O then.

a = 0 when t = 3

$v = \int a\,dt = 6t^2 - 36t + 7$ (using v = 7 when t = 0)

$v(3) = -47$

$x = \int v\,dt = 2t^3 - 18t^2 + 7t$ (x = 0 when t = 0)

$x(3) = -87$

Answer: (a) -47 m s^{-1} (b) -87 m

2 Disproof by counter-example (AQA, Edexcel, OCR, MEI, Eduqas)

Show by counter-example that the statement "$n^2 + n + 11$ is prime for all positive integers n" is false.

Try n = 10:

$10^2 + 10 + 11 = 121 = 11^2$

This is not prime.

Answer: n = 10 gives $121 = 11^2$, not prime

3 Comparing data sets from the large data set (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Summary statistics for the daily mean visibility (Dm) in the same month are:
Camborne: mean 23, standard deviation 3.6;
Leeming: mean 17, standard deviation 4.6.
Make two comparisons.

Means: Camborne has the higher mean (23 against 17), so on average the mean visibility is greater at Camborne

Standard deviations: Leeming is larger at Leeming (4.6 against 3.6)

so the readings at Leeming are more variable and those at Camborne are more consistent

Answer: Camborne higher on average; Leeming more variable

4 Chain rule and gradients (All boards)

$y = \sin^2(2x)$. Find $\frac{dy}{dx}$ and its exact value at $x = \frac{\pi}{6}$.

Let $u = \sin(2x)$: $y = u^2$, so $\frac{dy}{dx} = 2u^1 \times \frac{du}{dx}$

$\frac{du}{dx} = 2\cos(2x)$, giving $\frac{dy}{dx} = 4\sin(2x)\cos(2x)$

At $x = \frac{\pi}{6}$: $\sin(2x) = \frac{\sqrt{3}}{2}$ and $\cos(2x) = \frac{1}{2}$, so gradient $= \sqrt{3}$

Answer: $\frac{dy}{dx} = 4\sin(2x)\cos(2x)$; gradient $= \sqrt{3}$

5 Partial fractions: repeated factor (All boards)

Express $\frac{-4x^2 - 11x - 10}{(x-1)(x+4)^2}$ in the form $\frac{A}{(x-1)} + \frac{B}{(x+4)} + \frac{C}{(x+4)^2}$, and find A, B and C.

$-4x^2 - 11x - 10 \equiv A(x+4)^2 + B(x-1)(x+4) + C(x-1)$

x = 1: $-25 = A(5)^2$ so $A = -1$

x = −4: $-30 = C(-5)$ so $C = 6$

Coefficient of x^2: $-4 = A + B$ so $B = -3$

Answer: A = −1, B = −3, C = 6

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6 Connected particles (smooth table) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Particle A (4 kg) rests on a smooth table, joined by a light string over a smooth pulley at the edge to particle B (6 kg), hanging 1.2 m above the floor. Released from rest. Find (a) the tension, (b) the speed of B as it hits the floor. Take g = 9.8. Give answers to 3 s.f. if not exact.

B: $6g - T = 6a$
A: $T = 4a$
Adding: $6g = 10a$, so $a = 5.88$
(a) $T = 4 \times 5.88 = 23.52$
(b) $v^2 = 0 + 2 \times 5.88 \times 1.2 = 14.112$
Answer: (a) 23.52 N (b) 3.76 m s^{-1}

7 Equation after a sequence of transformations (All boards)

The curve with equation $y = x^2 - 6x - 5$ is transformed by a stretch, scale factor 2, parallel to the y-axis, followed by a translation of $\begin{pmatrix} -1 \\ -4 \end{pmatrix}$.

Find the equation of the image curve.

Stretch parallel to y: multiply by 2: $y = 2x^2 - 12x - 10$
Translation: replace x by (x + 1) and change y by −4: $y = 2x^2 - 8x - 24$
Answer: $y = 2x^2 - 8x - 24$

8 Foot of perpendicular and distance to a line (CIE (9709); Further Maths on other boards)

The line l has equation $\mathbf{r} = \begin{pmatrix} -5 \\ 1 \\ -3 \end{pmatrix} + t\begin{pmatrix} -3 \\ 2 \\ -3 \end{pmatrix}$.

Find the coordinates of the foot of the perpendicular from P(−5, −1, 3) to l, and the exact distance from P to l.

Foot F = A + t d: $\mathbf{PF} = \begin{pmatrix} -3t \\ 2t + 2 \\ -3t - 6 \end{pmatrix}$
$\mathbf{PF} \cdot \mathbf{d} = 0$ gives $22t + 22 = 0$, so $t = -1$
F = (−2, −1, 0); $\mathbf{PF} = \begin{pmatrix} 3 \\ 0 \\ -3 \end{pmatrix}$, so $|\mathbf{PF}| = \sqrt{18} = 3\sqrt{2}$
Answer: F(−2, −1, 0), distance $3\sqrt{2}$

9 Bayes' theorem in context (All boards)

0.5% of people have a disease. The screening test is positive for 90% of those who have it and for 4% of those who do not.
(a) Find the probability that a person tests positive.
(b) Given a positive test, find the probability that the person has the disease (3 s.f.).

(a) $P(+) = 0.005 \times 0.9 + 0.995 \times 0.04$
$= 0.0045 + 0.0398 = 0.0443$
(b) $P(\text{has it} \mid +) = \dfrac{0.005 \times 0.9}{0.0443}$
$= 0.102$
Answer: (a) 0.0443 (b) 0.102

10 Intersecting or skew lines (CIE (9709); Further Maths on other boards)

l_1: $\mathbf{r} = \begin{pmatrix} -1 \\ 1 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix}$

l_2: $\mathbf{r} = \begin{pmatrix} 4 \\ 1 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$

Determine whether l_1 and l_2 intersect. If they do, find the point of intersection.

Equate components: $-1 + 2\lambda = 4 + 1\mu$; $1 - 1\lambda = 1 + 2\mu$; $4 - 1\lambda = 1 - 1\mu$
Solving: $\lambda = 2$, $\mu = -1$, and all three equations are satisfied
Point of intersection = $(3, -1, 2)$
Answer: Intersect at $(3, -1, 2)$