Day 334 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Venn diagrams and conditional probability (All boards)

P(A) = 0.6, P(B) = 0.25 and P(A ∩ B) = 0.1.
Find
(a) P(A ∪ B)
(b) P(B | A)
(c) P(A | B′)

(a) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.6 + 0.25 - 0.1 = 0.75$

(b) $P(B \mid A) = \frac{P(A \cap B)}{P(A)} = \frac{0.1}{0.6} = \frac{1}{6}$

(c) $P(A \mid B') = \frac{P(A \cap B')}{P(B')} = \frac{0.6 - 0.1}{1 - 0.25} = \frac{2}{3}$

Answer: (a) 0.75 (b) $\frac{1}{6}$ (c) $\frac{2}{3}$

2 Composite functions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

f(x) = −2x + 1 and g(x) = x^2 + 6.
Solve gf(x) = 42.

gf(x) = g(−2x + 1) = $(-2x + 1)^2 + 6$
$(-2x + 1)^2 + 6 = 42$
$-2x + 1 = \pm 6$

Answer: $x = -\frac{5}{2}$ or $x = \frac{7}{2}$

3 Acceleration function of time (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves on the x-axis. At time t s its acceleration is (6t − 18) m s^{-2}. When t = 0 it is at O with velocity −8 m s^{-1}.
Find (a) its velocity when the acceleration is zero, (b) its displacement from O then.

a = 0 when t = 3
$v = \int a\,dt = 3t^2 - 18t - 8$ (using v = −8 when t = 0)
v(3) = −35
$x = \int v\,dt = t^3 - 9t^2 - 8t$ (x = 0 when t = 0)
x(3) = −78

Answer: (a) −35 m s^{-1} (b) −78 m

4 Selections with combinations (OCR, MEI, Eduqas, CIE)

A group of 9 boys and 6 girls. A team of 4 is chosen at random.
Find the probability that the team has exactly 1 of the girls.

Total teams = C(15, 4) = 1365
Favourable = C(6, 1) × C(9, 3) = 6 × 84 = 504

$P = \frac{504}{1365}$

$= \frac{24}{65}$

Answer: $\frac{24}{65}$

5 Double angle equations (All boards)

Solve cos 2θ − 2sin θ = −3
for −180° ≤ θ ≤ 180°.

cos 2θ ≡ 1 − $2\sin^2\theta$: $2\sin^2\theta + 2\sin\theta - 4 = 0$
Solve the quadratic: sin θ = 1 or sin θ = −2
sin θ = 1: θ = 90°
sin θ = −2 is impossible
Answer: θ = 90°

6 Final seconds of a fall (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A stone is dropped from rest from the top of a cliff. In the last 3 seconds before it hits the sea it falls 205.8 m. Find the height of the cliff. Take g = 9.8.

Let the total time of fall be t s. Then $s = 4.9t^2$ for the whole fall.
In the first $(t - 3)$ s the stone falls $4.9(t - 3)^2$.
$4.9t^2 - 4.9(t - 3)^2 = 205.8$
$4.9(6t - 9) = 205.8$, so $t = 8$
Height $= 4.9 \times 8^2 = 354.025$
Answer: 354.025 m

7 Partial fractions and integration (All boards)

(a) Express $\frac{-2x - 5}{(x + 4)^2}$ in the form $\frac{A}{x + 4} + \frac{B}{(x + 4)^2}$.

(b) Hence find the exact value of $\int_0^4 \frac{-2x - 5}{(x + 4)^2}\,dx$.

$-2x - 5 \equiv A(x + 4) + B$, so $A = -2$ and $B = 3$

$I = \left[-2\ln(x + 4) - \frac{3}{x + 4}\right]_0^4$

$= -2\ln\frac{8}{4} + 3\left(\frac{1}{4} - \frac{1}{8}\right)$

$= \frac{3 - 16\ln 2}{8}$

Answer: (a) $A = -2$, $B = 3$; (b) $\frac{3 - 16\ln 2}{8}$

8 Surds: equation with surd coefficients (AQA, Edexcel, OCR, MEI, Eduqas)

Solve $x(\sqrt{3} - 2) = 5 - 4\sqrt{3}$, writing x in the form $p + q\sqrt{3}$.

$x = \frac{5 - 4\sqrt{3}}{\sqrt{3} - 2}$

Multiply top and bottom by $\sqrt{3} + 2$.
Numerator: $(5 - 4\sqrt{3})(\sqrt{3} + 2) = -2 - 3\sqrt{3}$
Denominator: $3 - 4 = -1$
Answer: $x = 2 + 3\sqrt{3}$

9 Implicit differentiation: tangent to a curve (All boards)

A curve has equation $-x^2 + 4xy + 2y = 1$.
Show that P(−3, −1) lies on the curve and find the equation of the tangent at P in the form $ax + by + c = 0$ with integer a, b, c.

Differentiate (product rule on the xy term): $(-2x + 4y) + (4x + 2)\frac{dy}{dx} = 0$

At P(−3, −1): $2 + (-10)\frac{dy}{dx} = 0$, so $\frac{dy}{dx} = \frac{1}{5}$

Tangent: $y + 1 = \frac{1}{5}(x + 3)$, i.e. $x - 5y - 2 = 0$

Answer: $x - 5y - 2 = 0$

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10 Partial fractions and binomial expansion (All boards)

$f(x) = \frac{-11x + 5}{(1 - x)(1 - 3x)}$.

(a) Express f(x) in partial fractions.

(b) Hence expand f(x) up to x^2.

(c) State the range of x for which the expansion is valid.

(a) $f(x) = \frac{A}{(1 - x)} + \frac{B}{(1 - 3x)}$: A = 3, B = 2

(b) $3(1 - x)^{-1} = 3(1 + x + 1x^2 ...)$

Add the series for both fractions.

(c) Need |x| < 1 and |3x| < 1

Answer: (a) $\frac{3}{(1 - x)} + \frac{2}{(1 - 3x)}$ **(b)** $5 + 9x + 21x^2$ **(c)** $|x| < \frac{1}{3}$