Day 333 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Normal distribution probabilities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X, the height (cm), is normally distributed with mean 177 and standard deviation 7.
Find (a) P(X > 179.8) (b) P(179.8 < X < 185.4), each to 3 s.f.

(a) $z = \frac{179.8 - 177}{7} = 0.400$, P(Z > 0.400) = 0.345

(b) $z_1 = 0.400$, $z_2 = \frac{185.4 - 177}{7} = 1.20$

$P(179.8 < X < 185.4) = \Phi(1.20) - \Phi(0.400) = 0.230$

Answer: (a) 0.345 (b) 0.230

2 Small angle approximations: solving an equation (AQA, Edexcel, OCR, MEI, Eduqas)

θ is small and measured in radians. Use small angle approximations to show that
$5\sin 3\theta + \cos\theta = 9$
can be written as $a\theta^2 + b\theta + c = 0$ with integers a, b, c. Hence find the approximate solution with $0 < \theta < 0.8$, to 3 s.f.

$\sin 3\theta \approx 3\theta$, $\cos\theta \approx 1 - \frac{1}{2}\theta^2$

$5 \times 3\theta + 1(1 - \frac{1}{2}\theta^2) = 9$

Multiply by 2 and rearrange: $\theta^2 - 30\theta + 16 = 0$

$\theta = \frac{30 \pm \sqrt{836}}{2}$, so $\theta = 0.543$

Answer: a = 1, b = −30, c = 16; $\theta \approx 0.543$

3 Pulley, string goes slack (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Particles A (12 kg) and B (2 kg) hang on a light string over a smooth pulley, A being 1.5 m above the ground. Released from rest; A lands without rebounding. Find (a) the tension while both move, (b) the greatest height of B above its start. Take g = 9.8. Give answers to 3 s.f. if not exact.

A: $12g - T = 12a$; B: $T - 2g = 2a$
$a = 10g \div 14 = 7$, so $T = 2(g + a) = 33.6$
When A lands, B has risen 1.5 m with $v^2 = 2 \times 7 \times 1.5 = 21$

B then moves freely under gravity: extra height = $v^2 \div (2 \times 9.8) = \frac{15}{14}$

Total = $1.5 + \frac{15}{14}$

Answer: (a) 33.6 N (b) 2.57 m

4 Transformations of a turning point (All boards)

The curve y = f(x) has a maximum point at (5, 1).
Find the coordinates of the corresponding point on:
(a) y = f(x + 1)
(b) y = f(−x)
(c) y = −f(x)

(a) y = f(x + 1): (4, 1)
(b) y = f(−x): (−5, 1)
(c) y = −f(x): (5, −1)
A stretch or translation moves the point in the stated direction only.
Answer: (a) (4, 1); (b) (−5, 1); (c) (5, −1)

5 Regression line from summary data (AQA, Edexcel, OCR, MEI, Eduqas)

n = 10 pairs of (x, y): $\Sigma x = 135$, $\Sigma y = 358$, $\Sigma x^2 = 2525$, $\Sigma xy = 6230$.
(a) Find the equation of the regression line of y on x (3 s.f.).
(b) Estimate y when x = 18.

$S_{xx} = 2525 - \frac{135^2}{10} = 702$, $S_{xy} = 6230 - \frac{135 \times 358}{10} = 1400$

$b = \frac{S_{xy}}{S_{xx}} = 1.99$

$a = \bar{y} - b\,\bar{x} = 35.8 - (1.99)(13.5) = 8.95$

(b) $y = 8.95 + (1.99)(18) = 44.7$

Answer: (a) $y = 8.95 + 1.99x$ (b) 44.7

6 Asymptotes after transformation (All boards)

The curve y = f(x), where $f(x) = \frac{3}{x+1} + 2$, is translated by $\begin{pmatrix} -2 \\ 3 \end{pmatrix}$.

Write down the equations of the asymptotes of the new curve, and find where it crosses the x-axis.

Asymptotes of y = f(x): $x = -1$, $y = 2$

Translate by $\begin{pmatrix} -2 \\ 3 \end{pmatrix}$: $x = -3$, $y = 5$

New curve: $y = \frac{3}{x+3} + 5$

$y = 0$: $\frac{3}{x+3} = -5$, so $x + 3 = -\frac{3}{5}$

Answer: $x = -3$, $y = 5$; crosses x-axis at $\left(-\frac{18}{5}, 0\right)$

7 Reducing a relationship to a straight line (graph) (All boards)

The diagram shows the graph of ln y against x for $y = A\,e^{kx}$.
Find the exact values of A and k.

$\ln y = \ln A + kx$, a straight line with gradient k and intercept ln A

Intercept $\ln A = 1$, so $A = e^1$

Gradient $k = \frac{3 - 1}{2 - 0} = 1$

Answer: $A = e^1$, $k = 1$

8 Two balls meeting (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Ball A is dropped from rest 196 m above the ground. At the same instant ball B is projected upwards at 39.2 m s^{-1} from the ground directly below A. Find (a) the time when they collide, (b) the height of the collision. Take g = 9.8.

Let the time be t. Distance fallen by A: $\frac{1}{2}gt^2 = 4.9t^2$
Height of B above the ground: $39.2\,t - 4.9t^2$
They meet when the distances add up to 196: $4.9t^2 + (39.2\,t - 4.9t^2) = 196$
so $39.2\,t = 196$, $t = 5$
Height of B = $39.2 \times 5 - 4.9 \times 5^2 = 73.5$
Answer: (a) 5 s (b) 73.5 m

9 Implicit differentiation: horizontal tangents (All boards)

A curve has equation $4x^2 - 2xy + 3y^2 = 44$.
Find the coordinates of the points where the tangent is parallel to the x-axis.

$\frac{dy}{dx} = -\frac{8x - 2y}{-2x + 6y}$

Horizontal tangent: $8x - 2y = 0$, so $y = 4x$

Substitute: $44x^2 = 44$, so $x^2 = 1$ and $x = \pm 1$

Points: (1, 4) and (−1, −4)

Answer: (1, 4) and (−1, −4)

10 Sum of multiples in a range (All boards)

Find the sum of all the multiples of 8 that lie strictly between 113 and 358.

First multiple 120, last multiple 352, common difference 8.

Number of terms $n = \frac{352 - 120}{8} + 1 = 30$

$S = \frac{30}{2}(120 + 352) = 7080$

Answer: 7080