Day 332 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Arithmetic sequences: finding a and d (All boards)

The 4th term of an arithmetic sequence is 1 and the 8th term is 13.
Find the first term and common difference, and the sum of the first 15 terms.

$u_8 - u_4 = 4\ d = 13 - 1$ so $d = 3$

$a + 3\ d = 1$ so $a = -8$

$S_{15} = \frac{15}{2}(2 \times (-8) + 14 \times 3)$

Answer: $a = -8$, $d = 3$, $S_{15} = 195$

2 Histograms (All boards)

The histogram shows the waiting times, t (in minutes), of a sample.
(a) How many were in the class $50 \leq t < 70$?
(b) Estimate how many had $45 < t < 55$.

(a) Frequency = frequency density × width = 4 × 20 = 80
(b) 45 to 50: 4 × 5 = 20
50 to 55: 4 × 5 = 20
Total = 40
Answer: (a) 80 (b) 40

3 Transformations of a turning point (All boards)

The curve y = f(x) has a maximum point at (2, 5).
Find the coordinates of the corresponding point on:
(a) y = 3f(x)
(b) y = f(x) + 2
(c) y = f(4x)

(a) y = 3f(x): (2, 15)
(b) y = f(x) + 2: (2, 7)

(c) y = f(4x): $(\frac{1}{2}, 5)$

A stretch or translation moves the point in the stated direction only.

Answer: (a) (2, 15); (b) (2, 7); (c) $(\frac{1}{2}, 5)$

4 Scalar product, magnitude of a sum (CIE (9709); Further Maths on other boards)

$|a| = 5$, $|b| = 4$ and the angle between a and b is 150°.
Find the exact values of $a \cdot b$ and $|a + b|$.

$a \cdot b = |a||b|\cos 150° = 5 \times 4 \times -\frac{\sqrt{3}}{2} = -10\sqrt{3}$

$|a + b|^2 = |a|^2 + 2a \cdot b + |b|^2 = 25 + -20\sqrt{3} + 16 = 41 - 20\sqrt{3}$

$|a + b| = \sqrt{41 - 20\sqrt{3}}$

Answer: $a \cdot b = -10\sqrt{3}$, $|a + b| = \sqrt{41 - 20\sqrt{3}}$

5 Passing a point twice (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A stone is projected vertically upwards from the ground. It passes a ledge 1 s after projection and again 4 s after projection. Find (a) the speed of projection, (b) the height of the ledge, (c) the greatest height. Take g = 9.8.

Same height at t = 1 and t = 4, so the stone is at its highest point at t = 2.5
(a) At the top v = 0: $0 = u - 9.8 \times 2.5$, so $u = 24.5$
(b) $s = ut - 4.9t^2$ with t = 1: $s = 24.5 \times 1 - 4.9 \times 1^2 = 19.6$
(c) $v^2 = u^2 - 2gs$: $0 = 24.5^2 - 19.6s$, $s = 30.625$
Answer: (a) 24.5 m s^{-1} (b) 19.6 m (c) 30.625 m

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6 Implicit differentiation: products and cubes (All boards)

A curve has equation $3x^2y - y^3 - 4x = -2$.

Find $\frac{dy}{dx}$ in terms of x and y, and the gradient of the normal at (1, −2).

Differentiate: $6xy + 3x^2\frac{dy}{dx} - 3y^2\frac{dy}{dx} - 4 = 0$

$\frac{dy}{dx} = -\frac{6xy - 4}{3x^2 - 3y^2}$

At (1, −2): $\frac{dy}{dx} = -\frac{16}{9}$, so the normal has gradient $\frac{9}{16}$

Answer: $\frac{dy}{dx} = -\frac{6xy - 4}{3x^2 - 3y^2}$; normal gradient $\frac{9}{16}$

7 Circle through two points with centre on a line (AQA, Edexcel, OCR, MEI, Eduqas)

A circle passes through A(11, 9) and B(−6, 16).
Its centre lies on the line y = 4.
Find the equation of the circle.

Centre (h, 4) is equidistant from A and B.
$(h - 11)^2 + (-5)^2 = (h + 6)^2 + (-12)^2$
Solving gives h = −1, so centre (−1, 4)
$r^2 = (12)^2 + (5)^2 = 169$
Answer: $(x + 1)^2 + (y - 4)^2 = 169$

8 Equations with cosec x and cot x (All boards)

Solve $4\text{cosec}^2\,\theta - 18\cot\theta + 14 = 0$
for $0° < \theta < 360°$. Give answers to 1 d.p.

Use $\text{cosec}^2\,\theta \equiv 1 + \cot^2\theta$: $4\cot^2\theta - 18\cot\theta + 18 = 0$

$\cot\theta = \frac{3}{2}$ or $\cot\theta = 3$

$\cot\theta = \frac{3}{2}$ means $\tan\theta = \frac{2}{3}$: $\theta = 33.7°, 213.7°$

$\cot\theta = 3$ means $\tan\theta = \frac{1}{3}$: $\theta = 18.4°, 198.4°$

Answer: θ = 18.4°, 33.7°, 198.4°, 213.7°

9 Two balls meeting (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Ball A is dropped from rest 39.2 m above the ground. At the same instant ball B is projected upwards at 39.2 m s^{-1} from the ground directly below A. Find (a) the time when they collide, (b) the height of the collision. Take g = 9.8.

Let the time be t. Distance fallen by A: $\frac{1}{2}gt^2 = 4.9t^2$
Height of B above the ground: $39.2\,t - 4.9t^2$
They meet when the distances add up to 39.2: $4.9t^2 + (39.2\,t - 4.9t^2) = 39.2$
so $39.2\,t = 39.2$, $t = 1$
Height of B = $39.2 \times 1 - 4.9 \times 1^2 = 34.3$
Answer: (a) 1 s (b) 34.3 m

10 Type I and Type II errors (AQA, Edexcel, OCR, MEI, Eduqas)

X ~ B(35, p). H_0: p = 0.4 is tested against H_1: p > 0.4 using a 5% significance level.
(a) Find the critical region. (b) State the probability of a Type I error.
(c) Find the probability of a Type II error when p = 0.6 (3 s.f.).

(a) Largest/smallest region with probability ≤ 0.05 under H_0: $X \geq 20$
(b) Type I error = P(reject H_0 | p = 0.4) = 0.0300
(c) Type II error = P(not in critical region | p = 0.6)
$P(X \leq 19)$ when p = 0.6: 0.300
Answer: (a) $X \geq 20$ (b) 0.0300 (c) 0.300