Day 331 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Volumes of revolution (CIE (9709); Further Maths on other boards)

The region bounded by the curve $y = x^2 + 1$, the x-axis, $x = 0$ and $x = 3$ is rotated through 360° about the x-axis. Find the exact volume of the solid formed.

$y^2 = (x^2 + 1)^2 = x^4 + 2x^2 + 1$, so $V = \pi\int_0^3 y^2\,dx$

$= \pi \times \frac{348}{5}$

$V = \frac{348\pi}{5}$

Answer: $\frac{348\pi}{5}$

2 Sigma notation (All boards)

Evaluate $\sum_{r=6}^{16}(5r - 1)$.

Arithmetic series: first term 29, last term 79, 11 terms

$S = \frac{11}{2}(29 + 79) = 594$

Answer: 594

3 Index equations solved with logarithms (All boards)

Solve $3^{3x+2} = 2^{2x+1}$.

Give your answer to 3 significant figures.

Take logs: $(3x + 2)\ln 3 = (2x + 1)\ln 2$

Collect x terms: $x(3\ln 3 - 2\ln 2) = \ln 2 - 2\ln 3$

$x = \frac{\ln 2 - 2\ln 3}{3\ln 3 - 2\ln 2}$

$x = -0.788$

Answer: $x = -0.788$

4 Newton's second law with vectors (AQA, Edexcel, OCR, MEI, Eduqas)

A particle of mass 4 kg is acted on by forces $F_1 = (28\mathbf{i} - \mathbf{j})$ N and F_2. Its acceleration is $(4\mathbf{i} + \mathbf{j})$ m s^{-2}. Find (a) F_2, (b) the angle between F_2 and $\mathbf{i}$, to 1 d.p.

$F = ma$: $F_1 + F_2 = 4(4\mathbf{i} + \mathbf{j}) = 16\mathbf{i} + 4\mathbf{j}$

(a) $F_2 = (16\mathbf{i} + 4\mathbf{j}) - (28\mathbf{i} - \mathbf{j}) = -12\mathbf{i} + 5\mathbf{j}$

(b) $|F_2| = \sqrt{(-12^2 + 5^2)} = 13$

$\cos\theta = -12 \div 13$, so $\theta = 157.4°$

Answer: (a) $F_2 = (-12\mathbf{i} + 5\mathbf{j})$ N (b) 157.4°

5 Geometric distribution (CIE)

On each independent trial, the probability that a basketball player scores is $\frac{2}{5}$.

X is the number of trials up to and including the first success.

(a) Find P(X = 3) and E(X).

(b) Find the least m with $P(X \leq m) > 0.9$.

(a) $P(X = 3) = (\frac{3}{5})^2 \times \frac{2}{5} = \frac{18}{125}$; $E(X) = \frac{1}{p} = 2$

(b) $P(X \leq m) = 1 - (\frac{3}{5})^m > 0.9$

$(\frac{3}{5})^m < 0.1$, so $m > 4.51$

Least integer $m = 5$

Answer: (a) $\frac{18}{125}$, E(X) = 2 (b) m = 5

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6 Partial fractions: distinct linear factors (All boards)

Express $\frac{-11x + 8}{(3x - 3)(x - 4)}$ in partial fractions.

Let $\frac{-11x + 8}{(3x - 3)(x - 4)} \equiv \frac{A}{(3x - 3)} + \frac{B}{(x - 4)}$

$-11x + 8 \equiv A(x - 4) + B(3x - 3)$

Substitute $x = 1$: $A = 1$

Substitute $x = 4$: $B = -4$

Answer: $\mathbf{\frac{1}{(3x - 3)} + \frac{-4}{(x - 4)}}$

7 Continuous random variable: finding k and E(X) (CIE)

The random variable X has probability density function $f(x) = kx(2 - x)$ for $0 \leq x \leq 2$, and 0 otherwise.
(a) Find k. (b) Find E(X). (c) Find $P(X > 1)$.

(a) $\int_0^2 f(x)\,dx = 1$ gives $\frac{4}{3}k = 1$, so $k = \frac{3}{4}$

(b) $E(X) = \int x\,f(x)\,dx = 1$

(c) $P(X > 1) = \int_1^2 f(x)\,dx = \frac{1}{2}$

Answer: (a) $k = \frac{3}{4}$ (b) 1 (c) $\frac{1}{2}$

8 Reducing a relationship to a straight line (graph) (All boards)

The diagram shows the graph of lg y against lg x for $y = A x^n$.
Find the value of n and the value of A.

$\lg y = \lg A + n \lg x$: gradient n, intercept $\lg A$

$n = \frac{13 - 7}{3 - 1} = 3$

At (1, 7): $\lg A = 7 - 3 \times 1 = 4$, so $A = 10^4 = 10000$

Answer: n = 3, A = 10000

9 Arithmetic modelling (All boards)

A charity raises £400 in its first year and in each later year raises £50 more than in the previous year.
(a) How much does it raise in year 13? (b) In which year does the total raised first exceed £5000?

(a) $u_{13} = 400 + 12 \times 50 = 1000$

(b) $S_m = \frac{m}{2}(800 + (m - 1) \times 50) > 5000$

$S_8 = 4600 < 5000$ and $S_9 = 5400 > 5000$

Answer: (a) £1000 (b) year 9

10 Position vector and velocity (AQA, Edexcel, OCR, MEI, Eduqas)

A particle P has position vector $\mathbf{r} = (2t^3 - 3t)\mathbf{i} + (t^2 - 2t)\mathbf{j}$ metres at time t s. Find (a) the speed of P when $t = 2$, (b) the value of t when P is moving parallel to $\mathbf{i}$. Give answers to 3 s.f. if not exact.

$\mathbf{v} = d\mathbf{r}/dt = (6t^2 - 3)\mathbf{i} + (2t - 2)\mathbf{j}$

(a) $t = 2$: $\mathbf{v} = 21\mathbf{i} + 2\mathbf{j}$

Speed $= \sqrt{(21^2 + 2^2)} = \sqrt{445}$

(b) Parallel to $\mathbf{i}$ means the $\mathbf{j}$ component is zero: $2t - 2 = 0$, so $t = 1$

Answer: (a) 21.1 m s^{-1} (b) t = 1