Day 330 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Work-energy on a rough slope (CIE)

A 3 kg block is released from rest on a rough plane inclined at α, $\tan\alpha = \frac{3}{4}$, $\mu = 0.25$, and slides 2 m down a line of greatest slope. Use work and energy to find (a) the work done against friction, (b) its speed after 2 m. Take $g = 10$.

$R = 3g\cos\alpha = 24$ N, $F = \mu R = 6$ N
(a) Work done against friction $= F \times d = 6 \times 2$
(b) Loss of PE $= 3g \times 2\sin\alpha = 36$ J
Work-energy: $\frac{1}{2}\times 3\times v^2 = 36 - 12 = 24$
$v^2 = 16$
Answer: (a) 12 J (b) 4 m s^{-1}

2 Newton-Raphson method (AQA, Edexcel, OCR, MEI, Eduqas)

$f(x) = x^2 - 7$. Show that the Newton-Raphson formula gives $x_{n+1} = \frac{1}{2}\left(x_n + \frac{7}{x_n}\right)$, and use it with $x_0 = 2$ to find x_2 as an approximation to $\sqrt{7}$, to 4 d.p.

$f'(x) = 2x$, so $x_{n+1} = x_n - \frac{x_n^2 - 7}{2x_n} = \frac{1}{2}\left(x_n + \frac{7}{x_n}\right)$

$x_1 = \frac{1}{2}\left(2 + \frac{7}{2}\right) = \frac{11}{4}$
$x_2 = 2.6477$
Answer: $x_2 = 2.6477$

3 Comparing data sets from the large data set (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Summary statistics for the daily mean temperature (°C) in the same month are:
Perth: mean 26, standard deviation 4.8;
Jacksonville: mean 8, standard deviation 1.9.
Make two comparisons.

Means: Perth has the higher mean (26 against 8), so on average the mean temperature is greater at Perth
Standard deviations: Perth is larger at Perth (4.8 against 1.9)
so the readings at Perth are more variable and those at Jacksonville are more consistent
Answer: Perth higher on average; Perth more variable

4 Cubic inequalities (All boards)

Given that $(x + 3)$ is a factor of $f(x) = x^3 - 2x^2 - 11x + 12$, solve $f(x) \geq 0$.

Factorise: $f(x) = (x + 3)(x - 1)(x - 4)$
Roots at $x = -3, 1, 4$
Sketch the cubic (positive x^3 coefficient: negative, positive, negative, positive)
Read off where $f(x) \geq 0$.
Answer: $-3 \leq x \leq 1$ or $x \geq 4$

5 Coding mean and standard deviation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

For 50 values of x, the coded variable $y = \frac{x - 100}{4}$ gives

$\Sigma y = 150$ and $\Sigma y^2 = 900$.
Find the mean and standard deviation of x.

$\bar{y} = \frac{150}{50} = 3$

$\sigma_y^2 = \frac{900}{50} - 3^2 = 9$, so $\sigma_y = 3$

$x = 4y + 100$, so mean $= 4 \times 3 + 100 = 112$
Standard deviation $= 4 \times 3 = 12$
Answer: mean 112, sd 12

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6 Forces in equilibrium with vectors (AQA, Edexcel, OCR, MEI, Eduqas)

Forces $F_1 = (-4\mathbf{i} - 9\mathbf{j})$ N, $F_2 = (-3\mathbf{i} - 4\mathbf{j})$ N and $F_3 = (a\mathbf{i} + b\mathbf{j})$ N keep a 2 kg particle in equilibrium. Find (a) a and b. F_2 is then removed; (b) find the magnitude of the acceleration.

(a) Equilibrium: $F_1 + F_2 + F_3 = 0$

$\mathbf{i}$: $-4 + (-3) + a = 0$, so $a = 7$

$\mathbf{j}$: $-9 + (-4) + b = 0$, so $b = 13$

(b) The resultant is now $F_1 + F_3 = -F_2$, of magnitude $\sqrt{(-3^2 + -4^2)} = 5$ N

$a = 5 \div 2$

Answer: (a) a = 7, b = 13 (b) 2.5 m s^{-2}

7 Parametric differentiation: tangent and normal (All boards)

A curve has parametric equations $x = 3t - 1$, $y = t^3$.

Find $\frac{dy}{dx}$ in terms of t, and the equation of the normal at the point where $t = -\frac{1}{2}$, in the form $ax + by + c = 0$ (integers).

$\frac{dy}{dx} = \frac{dy}{dt \div dx/dt} = t^2$

$t = -\frac{1}{2}$: point $(-\frac{5}{2}, -\frac{1}{8})$, gradient $\frac{1}{4}$

Normal gradient $= -1 \div \frac{1}{4} = -4$, so normal: $32x + 8y + 81 = 0$

Answer: $\frac{dy}{dx} = t^2$; $32x + 8y + 81 = 0$

8 Quadratic from its graph (All boards)

The diagram shows the curve with equation $y = f(x)$, where f is a quadratic.
The turning point is marked, and the curve crosses the y-axis as shown.
Find f(x), giving it in the form $ax^2 + bx + c$.

Turning point $(-3, 8)$: $f(x) = a(x + 3)^2 + 8$

$f(0) = -19$: $a(-3)^2 + 8 = -19$

$a = -3$, then expand

Answer: f(x) = $-3x^2 - 18x - 19$

9 Proof by contradiction (AQA, Edexcel, OCR, MEI, Eduqas)

Prove by contradiction that there are no integers m and n such that $10m + 15n = 117$.

Assume $10m + 15n = 117$ for integers m, n.

Then $5(2m + 3n) = 117$, so 5 divides 117.

But 117 is not a multiple of 5. Contradiction.

Answer: no such integers exist

10 Intersecting or skew lines (CIE (9709); Further Maths on other boards)

l_1: $\mathbf{r} = \begin{pmatrix} -5 \\ 0 \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$

l_2: $\mathbf{r} = \begin{pmatrix} 13 \\ 3 \\ 9 \end{pmatrix} + \mu \begin{pmatrix} -3 \\ 1 \\ -3 \end{pmatrix}$

Determine whether l_1 and l_2 intersect. If they do, find the point of intersection.

Equate components: $-5 + 3\lambda = 13 - 3\mu$; $0 + 2\lambda = 3 + 1\mu$; $-3 + 1\lambda = 9 - 3\mu$

Solving: $\lambda = 3$, $\mu = 3$, and all three equations are satisfied

Point of intersection = (4, 6, 0)

Answer: Intersect at (4, 6, 0)