Day 329 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Exponential growth and decay modelling (All boards)

A car costs £18000 new. Its value falls by 12% each year, so after t years its value is £V.
(a) Write down V in terms of t. (b) Find, to 3 s.f., the time when the value is £6000.

$V = 18000 \times 0.88^t$

$18000 \times 0.88^t = 6000$ gives $0.88^t = \frac{1}{3}$

$t = \frac{\ln(1}{3)/\ln 0.88} = 8.59$ years

Answer: (a) $V = 18000 \times 0.88^t$ (b) $t = 8.59$ years

2 Passing a point twice (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A stone is projected vertically upwards from the ground. It passes a ledge 1 s after projection and again 5 s after projection. Find (a) the speed of projection, (b) the height of the ledge, (c) the greatest height. Take g = 9.8.

Same height at t = 1 and t = 5, so the stone is at its highest point at t = 3
(a) At the top v = 0: $0 = u - 9.8\times3$, so $u = 29.4$
(b) $s = ut - 4.9t^2$ with t = 1: $s = 29.4\times1 - 4.9\times1^2 = 24.5$
(c) $v^2 = u^2 - 2gs$: $0 = 29.4^2 - 19.6s$, $s = 44.1$

Answer: (a) 29.4 m s^{-1} (b) 24.5 m (c) 44.1 m

3 Histograms (All boards)

The histogram shows the distances thrown, d (in metres), of a sample.
(a) How many were in the class $50 \le d < 70$?
(b) Estimate how many had $45 < d < 120$.

(a) Frequency = frequency density × width = 6 × 20 = 120
(b) 45 to 50: 1 × 5 = 5
whole bar 50 to 70: 120
whole bar 70 to 100: 90
100 to 120: 1 × 20 = 20
Total = 235
Answer: (a) 120 (b) 235

4 Index equations solved with logarithms (All boards)

Solve $7^{3x-2} = 3^{3x-3}$.
Give your answer to 3 significant figures.

Take logs: $(3x - 2)\ln 7 = (3x - 3)\ln 3$
Collect x terms: $x(3\ln 7 - 3\ln 3) = -3\ln 3 + 2\ln 7$
$x = \frac{-3\ln 3 + 2\ln 7}{3\ln 7 - 3\ln 3}$
$x = 0.234$
Answer: x = 0.234

5 Partial fractions: distinct linear factors (All boards)

Express $\frac{-15x - 10}{(x + 1)(2x + 1)}$ in partial fractions.

Let $\frac{-15x - 10}{(x + 1)(2x + 1)} \equiv \frac{A}{(x + 1)} + \frac{B}{(2x + 1)}$
$-15x - 10 \equiv A(2x + 1) + B(x + 1)$
Substitute $x = -1$: $A = -5$
Substitute $x = -\frac{1}{2}$: $B = -5$
Answer: $\frac{-5}{(x + 1)} + \frac{-5}{(2x + 1)}$

6 Car and trailer (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A 900 kg car tows a 500 kg trailer by a light tow-bar on a horizontal road. Driving force 1500 N; resistances 600 N (car), 200 N (trailer). Find (a) the acceleration and the tow-bar tension. At 12 m s^{-1} the tow-bar breaks. (b) How far does the trailer then travel?

Whole system: $1500 - 600 - 200 = 1400\,a$, so $a = 0.5$

Trailer: $T - 200 = 500\,a$, so $T = 200 + 500\times0.5 = 450$

(b) After the break the trailer decelerates at $200 \div 500 = 0.4$ m s^{-2}

$0 = 12^2 - 2\times0.4\times s$, so $s = 180$

Answer: (a) 0.5 m s^{-2}, 450 N (b) 180 m

7 Foot of a perpendicular (All boards)

The line l has equation $x - 3y = -2$.

(a) Find the coordinates of the foot of the perpendicular from P$(-6, -4)$ to l.

(b) Hence find the shortest distance from P to l, in exact form.

Gradient of l $= \frac{1}{3}$, so perpendicular gradient $= -3$

Perpendicular through P: $-3x - y = 22$

Solve with l: $x = -\frac{34}{5}$, $y = -\frac{8}{5}$

Distance$^2 = (-6 - (-\frac{34}{5}))^2 + (-4 - (-\frac{8}{5}))^2 = \frac{32}{5}$

Answer: (a) $(-\frac{34}{5}, -\frac{8}{5})$ (b) $\frac{4\sqrt{10}}{5}$

8 R-form and solving a sin x + b cos x = c (All boards)

(a) Express $21\cos x - 20\sin x$ in the form $R\cos(x + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. Give α to 1 d.p.

(b) Solve $21\cos x - 20\sin x = -14$ for $0° \le x < 360°$, giving answers to 1 d.p.

$R^2 = 21^2 + 20^2 = 841$, so $R = 29$; $\tan\alpha = \frac{20}{21}$ gives $\alpha = 43.6°$

$29\cos(x + 43.6°) = -14$, so $\cos(x + 43.6°) = \frac{-14}{29}$

Principal value 118.9°, then use the symmetry of the cosine graph

$x = 75.3°, 197.5°$

Answer: (a) R = 29, α = 43.6° (b) x = 75.3°, 197.5°

9 Circle touching an axis (diagram) (AQA, Edexcel, OCR, MEI, Eduqas)

The diagram shows a circle that touches the x-axis at A(1, 0) and passes through B$(-4, 5)$.

Find the equation of the circle.

Touches the x-axis at A, so the centre is $(1, r)$ and the radius is r.

B on the circle: $(-5)^2 + (5 - r)^2 = r^2$

$25 + 25 - 10r = 0$ so $r = 5$

Answer: $(x - 1)^2 + (y - 5)^2 = 25$

10 Type I and Type II errors (AQA, Edexcel, OCR, MEI, Eduqas)

$X \sim B(22, p)$. H_0: $p = 0.3$ is tested against H_1: $p > 0.3$ using a 5% significance level.

(a) Find the critical region. (b) State the probability of a Type I error.

(c) Find the probability of a Type II error when $p = 0.5$ (3 s.f.).

(a) Largest/smallest region with probability ≤ 0.05 under H_0: $X \ge 11$

(b) Type I error = P(reject H_0 | $p = 0.3$) = 0.0387

(c) Type II error = P(not in critical region | $p = 0.5$)

$P(X \le 10)$ when $p = 0.5$: 0.416

Answer: (a) $X \ge 11$ (b) 0.0387 (c) 0.416