Day 327 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Change of sign to locate a root (All boards)

$f(x) = e^x - 2x - 8$.
Show that the equation $f(x) = 0$ has a root α in the interval $[2, 3]$.

$f(x)$ is continuous on the interval
$f(2) = -4.61 < 0$
$f(3) = 6.09 > 0$
A change of sign, so a root lies in $[2, 3]$
Answer: Sign change: $f(2) = -4.61$, $f(3) = 6.09$

2 Comparing data sets from the large data set (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Summary statistics for the daily mean windspeed (knots) in the same month are:
Heathrow: mean 9, standard deviation 5.1;
Leuchars: mean 22, standard deviation 0.9.
Make two comparisons.

Means: Leuchars has the higher mean (22 against 9), so on average the mean windspeed is greater at Leuchars
Standard deviations: Heathrow is larger at Heathrow (5.1 against 0.9)
so the readings at Heathrow are more variable and those at Leuchars are more consistent
Answer: Leuchars higher on average; Heathrow more variable

3 Surds: sums of fractions (AQA, Edexcel, OCR, MEI, Eduqas)

Show that $\frac{3}{\sqrt{11} - 1} + \frac{4}{\sqrt{11} + 1}$ can be written as $p + q\sqrt{11}$, and find p and q.

Common denominator $(\sqrt{11} - 1)(\sqrt{11} + 1) = 11 - 1 = 10$
Numerator $= 3(\sqrt{11} + 1) + 4(\sqrt{11} - 1)$
$= -1 + 7\sqrt{11}$
Divide by 10.
Answer: $p = -\frac{1}{10}$, $q = \frac{7}{10}$

4 Lift and cable tension (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A lift of mass 300 kg carries a person of mass 50 kg. The cable tension is 3640 N and the lift is accelerating. Find (a) the magnitude and direction of the acceleration, (b) the force of the floor on the person. Take $g = 9.8$.

Whole system (mass 350 kg), taking upwards as positive: $T - 350\,g = 350\,a$
$3640 - 3430 = 350\,a$, so $a = 0.6$
(a) a is positive, so the acceleration is $0.6\ \text{m s}^{-2}$ upwards
(b) Person: $R - 50\,g = 50\,a$, so $R = 50(9.8 + 0.6)$
Answer: (a) $0.6\ \text{m s}^{-2}$ upwards (b) 520 N

5 Scalar product, magnitude of a sum (CIE (9709); Further Maths on other boards)

$|\mathbf{a}| = 6$, $|\mathbf{b}| = 6$ and the angle between $\mathbf{a}$ and $\mathbf{b}$ is 120°.
Find the exact values of $\mathbf{a}\cdot\mathbf{b}$ and $|\mathbf{a} + \mathbf{b}|$.

$\mathbf{a}\cdot\mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos 120° = 6 \times 6 \times -\frac{1}{2} = -18$

$|\mathbf{a} + \mathbf{b}|^2 = |\mathbf{a}|^2 + 2\mathbf{a}\cdot\mathbf{b} + |\mathbf{b}|^2 = 36 + -36 + 36 = 36$
$|\mathbf{a} + \mathbf{b}| = 6$
Answer: $\mathbf{a}\cdot\mathbf{b} = -18$, $|\mathbf{a} + \mathbf{b}| = 6$

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6 Non-uniform rod and equal tensions (AQA, Edexcel, OCR, MEI)

A non-uniform rod AB (length 2 m, mass 6 kg) has its centre of mass at G, AG = 0.5 m. It hangs horizontally from vertical strings at A and B. A particle of mass m kg is fixed at P, AP = 1.5 m, making the tensions equal. Find (a) m, (b) the tension. Take g = 9.8.

Equal tensions T: 2T = (6 + m)g, so T = ½(6 + m)g
Moments about A: T×2 = 6 g×0.5 + m g×1.5
½(6 + m)×2 = 3 + 1.5 m → 6 + 1 m = 3 + 1.5 m
m (1.5 − 1) = 6 − 3, so m = 6
(b) T = ½(6 + 6)×9.8
Answer: (a) m = 6 (b) 58.8 N

7 Circle through two points with centre on a line (AQA, Edexcel, OCR, MEI, Eduqas)

A circle passes through A(5, 12) and B(−1, −6).
Its centre lies on the line $y = -x + 3$.
Find the equation of the circle.

Centre $(h, -h + 3)$ is equidistant from A and B.
$(h - 5)^2 + (-h - 9)^2 = (h + 1)^2 + (-h + 9)^2$
Solving gives h = −1, so centre (−1, 4)
$r^2 = (6)^2 + (8)^2 = 100$
Answer: $(x + 1)^2 + (y - 4)^2 = 100$

8 Equations with cosec x and cot x (All boards)

Solve $4\operatorname{cosec}^2 x - 4\cot x - 52 = 0$
for 0° < x < 360°. Give answers to 1 d.p.

Use $\operatorname{cosec}^2 x \equiv 1 + \cot^2 x$: $4\cot^2 x - 4\cot x - 48 = 0$
cot x = −3 or cot x = 4
cot x = −3 means $\tan x = -\frac{1}{3}$: x = 161.6°, 341.6°
cot x = 4 means $\tan x = \frac{1}{4}$: x = 14.0°, 194.0°
Answer: x = 14.0°, 161.6°, 194.0°, 341.6°

9 Estimating mean and standard deviation (All boards)

The table summarises the mass, m grams, of eggs.
Estimate the mean and standard deviation (3 s.f.).

Midpoints: 50, 70, 90, 110, 130, 150 (n = 51)
$\Sigma fx = 4930$, so mean = $\frac{4930}{51} = 96.7$
$\Sigma fx^2 = 526700$
sd = $\sqrt{\frac{526700}{51} - (96.7)^2} = 31.4$
Answer: mean 96.7, sd 31.4

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10 Binomial expansion: approximating a square root (All boards)

(a) Expand $(1 + 3x)^{\frac{1}{2}}$ in ascending powers of x up to x^2.

(b) Use x = 0.01 to estimate $\sqrt{103}$ to 4 decimal places.

$(1 + y)^{\frac{1}{2}} = 1 + \frac{1}{2}y - \frac{1}{8}y^2$, y = 3x

(a) $1 + \frac{3}{2}x - \frac{9}{8}x^2$

$x = \frac{1}{100}$: $103 = 10^2(1 + \frac{3}{100})$ so $\sqrt{103} = 10 \times (1 + \frac{3}{100})^{\frac{1}{2}}$

$\approx 10 \times (1 + \frac{3}{200} - \frac{9}{80000}) = 10.148875$

Answer: (a) $1 + \frac{3}{2}x - \frac{9}{8}x^2$ (b) 10.1489