Day 326 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Double angle equations (All boards)

Solve $\cos 2\theta + \sin\theta = -2$
for $0° \leq \theta \leq 360°$.

$\cos 2\theta \equiv 1 - 2\sin^2\theta$: $2\sin^2\theta - \sin\theta - 3 = 0$

Solve the quadratic: $\sin\theta = -1$ or $\sin\theta = \frac{3}{2}$

$\sin\theta = -1$: $\theta = 270°$

$\sin\theta = \frac{3}{2}$ is impossible

Answer: $\theta = 270°$

2 Circle: centre, radius and position of a point (AQA, Edexcel, OCR, MEI, Eduqas)

A circle has equation $x^2 + y^2 - 6x + 12y + 16 = 0$.
(a) Find the centre and radius of the circle.
(b) Determine whether P(3, −6) lies inside or outside the circle.

Complete the square: $(x - 3)^2 + (y + 6)^2 = 29$

Centre (3, −6), radius $\sqrt{29}$

Distance^2 from centre to P = $0^2 + 0^2 = 0$

Compare with $r^2 = 29$

Answer: (a) centre (3, −6), radius $\sqrt{29}$ (b) P is inside the circle

3 Forces in equilibrium with vectors (AQA, Edexcel, OCR, MEI, Eduqas)

Forces $F_1 = (\mathbf{i} + 8\mathbf{j})$ N, $F_2 = (-15\mathbf{i} - 8\mathbf{j})$ N and $F_3 = (a\mathbf{i} + b\mathbf{j})$ N keep a 5 kg particle in equilibrium. Find (a) a and b. F_2 is then removed; (b) find the magnitude of the acceleration.

(a) Equilibrium: $F_1 + F_2 + F_3 = 0$
i: $1 + (-15) + a = 0$, so $a = 14$
j: $8 + (-8) + b = 0$, so $b = 0$
(b) The resultant is now $F_1 + F_3 = -F_2$, of magnitude $\sqrt{(-15^2 + -8^2)} = 17$ N
$a = 17 \div 5$

Answer: (a) a = 14, b = 0 (b) 3.4 m s^{-2}

4 Scalar product, magnitude of a sum (CIE (9709); Further Maths on other boards)

$|a| = 2$, $|b| = 7$ and the angle between a and b is 135°.
Find the exact values of $a\cdot b$ and $|a + b|$.

$a\cdot b = |a||b|\cos 135° = 2 \times 7 \times -\frac{\sqrt{2}}{2} = -7\sqrt{2}$

$|a + b|^2 = |a|^2 + 2a\cdot b + |b|^2 = 4 + -14\sqrt{2} + 49 = 53 - 14\sqrt{2}$

$|a + b| = \sqrt{53 - 14\sqrt{2}}$

Answer: $\mathbf{a\cdot b = -7\sqrt{2}}$, $\mathbf{|a + b| = \sqrt{53 - 14\sqrt{2}}}$

5 Binomial expansion: rational powers (All boards)

Expand $(1 + x)^{\frac{1}{2}}$ in ascending powers of x up to and including the term in x^2.
State the range of values of x for which the expansion is valid.

$(1 + y)^n = 1 + ny + \frac{n(n-1)}{2!}y^2 + \frac{n(n-1)(n-2)}{3!}y^3 + \ldots$ with $n = \frac{1}{2}$, $y = x$

Coefficients of 1, x, x^2: $1, \frac{1}{2}, -\frac{1}{8}$

Valid when $|x| < 1$, i.e. $|x| < 1$

Answer: $\mathbf{1 + \frac{1}{2}x - \frac{1}{8}x^2}$; $\mathbf{|x| < 1}$

6 Estimating mean and standard deviation (All boards)

The table summarises the height, h cm, of seedlings.
Estimate the mean and standard deviation (3 s.f.).

Midpoints: 25, 35, 45, 55, 65 (n = 51)

$\Sigma fx = 2355$, so mean $= \frac{2355}{51} = 46.2$

$\Sigma fx^2 = 123475$

sd $= \sqrt{\frac{123475}{51} - (46.2)^2} = 17.0$

Answer: mean 46.2, sd 17.0

7 Projectile from a cliff top (AQA, Edexcel, OCR, MEI, Eduqas)

A stone is projected from a cliff top 8.1 m above the sea at 15 m s^{-1}, at angle α above the horizontal, $\tan\alpha = \frac{4}{3}$. Find (a) the time to reach the sea, (b) the horizontal distance travelled, (c) the speed on impact. Take g = 9.8. Give answers to 3 s.f. if not exact.

Horizontal component 9, vertical component 12 (upwards)
(a) $-8.1 = 12\,t - 4.9t^2$, so $4.9t^2 - 12\,t - 8.1 = 0$
$t = [12 + \sqrt{302.76}] \div 9.8 = 3$ (taking the positive root)
(b) Distance = 9×3
(c) Vertical speed = $12 - 9.8\times3 = -17.4$; speed = $\sqrt{(9^2 + 17.4^2)}$

Answer: (a) 3 s (b) 27 m (c) 19.6 m s^{-1}

8 Binomial expansion: finding n and k (All boards)

In the expansion of $(1 + kx)^n$, where n is a positive integer, the coefficient of x is 12 and the coefficient of x^2 is 66.
Find n and k.

$nk = 12$ and $\frac{n(n-1)}{2}k^2 = 66$

$(n-1)k = \frac{2\times66}{12} = 11$ (dividing the second equation by the first)

$n\,k - (n-1)\,k = k = 12 - 11 = 1$

$n = \frac{12}{1} = 12$

Answer: n = 12, k = 1

9 Intersecting or skew lines (CIE (9709); Further Maths on other boards)

l_1: $\mathbf{r} = \begin{pmatrix}3\\1\\3\end{pmatrix} + \lambda\begin{pmatrix}-3\\0\\-2\end{pmatrix}$

l_2: $\mathbf{r} = \begin{pmatrix}-9\\-5\\-3\end{pmatrix} + \mu\begin{pmatrix}2\\3\\1\end{pmatrix}$

Determine whether l_1 and l_2 intersect. If they do, find the point of intersection.

Equate components: $3 - 3\lambda = -9 + 2\mu$; $1 + 0\lambda = -5 + 3\mu$; $3 - 2\lambda = -3 + 1\mu$
Solving two of the equations gives a unique (λ, μ)
which does not satisfy the third, so the lines do not meet

Answer: The lines do not intersect (skew)

10 Type I and Type II errors (AQA, Edexcel, OCR, MEI, Eduqas)

$X \sim B(32, p)$. H_0: $p = 0.3$ is tested against H_1: $p < 0.3$ using a 5% significance level.

(a) Find the critical region. (b) State the probability of a Type I error.

(c) Find the probability of a Type II error when $p = 0.15$ (3 s.f.).

(a) Largest/smallest region with probability ≤ 0.05 under H_0: $X \leq 4$

(b) Type I error = P(reject H_0 | $p = 0.3$) = 0.0189

(c) Type II error = P(not in critical region | $p = 0.15$)

$P(X \geq 5)$ when $p = 0.15$: 0.536

Answer: (a) $X \leq 4$ (b) 0.0189 (c) 0.536