Day 324 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Vertical motion under gravity (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A ball is thrown vertically upwards at 29.4 m s^{-1} from a point h m above horizontal ground, and hits the ground 7 s later. Find (a) h, (b) the greatest height above the ground. Take g = 9.8.

(a) Take upwards as positive: s = −h, u = 29.4, a = −9.8, t = 7
$-h = 29.4\times7 - 4.9\times7^2$, so h = 34.3
(b) At the highest point v = 0: $0 = 29.4^2 - 2\times9.8\times s$
s = 44.1 m above the point of projection
Greatest height above the ground = 34.3 + 44.1
Answer: (a) 34.3 m (b) 78.4 m

2 Composite functions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = x^2 + 2$ and $g(x) = 2x - 3$.
Solve fg(x) = gf(x).

$fg(x) = (2x - 3)^2 + 2 = 4x^2 - 12x + 11$
$gf(x) = 2(x^2 + 2) - 3 = 2x^2 + 1$
$4x^2 - 12x + 11 = 2x^2 + 1$ so $2x^2 - 12x + 10 = 0$
Answer: x = 1 or x = 5

3 Normal distribution probabilities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X, the length (mm), is normally distributed with mean 83 and standard deviation 2.
Find (a) P(X > 78.2) (b) P(78.2 < X < 79.6), each to 3 s.f.

(a) $z = \frac{78.2 - 83}{2} = -2.40$, $P(Z > -2.40) = 0.992$

(b) $z_1 = -2.40$, $z_2 = \frac{79.6 - 83}{2} = -1.70$
$P(78.2 < X < 79.6) = \Phi(-1.70) - \Phi(-2.40) = 0.0364$
Answer: (a) 0.992 (b) 0.0364

4 Inverse trigonometric functions (AQA, Edexcel, OCR, MEI, Eduqas)

$f(x) = 3\arcsin(2x)$.
State the domain and range of f, and find $f^{-1}(x)$.

arcsin has domain −1 to 1, so $-1 \le 2x \le 1$
Range of arcsin is $-\frac{\pi}{2}$ to $\frac{\pi}{2}$, so f(x) runs from $-\frac{3\pi}{2}$ to $\frac{3\pi}{2}$
$y = 3\arcsin(2x)$ gives $\sin\frac{y}{3} = 2x$, so $f^{-1}(x) = \frac{1}{2}\sin\frac{x}{3}$
Answer: domain $-\frac{1}{2} \le x \le \frac{1}{2}$, range $-\frac{3\pi}{2} \le f(x) \le \frac{3\pi}{2}$, $f^{-1}(x) = \frac{1}{2}\sin\frac{x}{3}$

5 Sigma notation (All boards)

Given that $\sum_{r=1}^{n}(4r - 2) = 162$, find the value of n.

$\sum_{r=1}^{n}(4r - 2) = 4 \times \frac{n(n+1)}{2} - 2n$
$2n^2 = 162$
$4n^2 - 324 = 0$
Solve and reject the negative root: n = 9
Answer: 9

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6 Exact definite integrals of standard functions (All boards)

Find the exact value of

$\int_0^{\ln 3} (5e^{2x} + 5e^x)\,dx$.

Integrate: $[\frac{5}{2}e^{2x} + 5e^x]$

Substitute the limits and subtract: 30

Answer: 30

7 Circle and line: tangent condition (AQA, Edexcel, OCR, MEI, Eduqas)

The line $5x + 12y = k$ is a tangent to the circle $(x + 2)^2 + (y + 1)^2 = 9$.

Find the two possible values of k.

Centre $(-2, -1)$, radius 3

Distance from centre to the line = radius:

$\frac{|-22 - k|}{13} = 3$

$|-22 - k| = 39$

Answer: $k = -61$ or $k = 17$

8 Pulled at an angle on rough ground (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A 4 kg box is pulled along rough horizontal ground, $\mu = 0.3$, by a rope at angle θ above the horizontal, where $\tan\theta = \frac{5}{12}$. The tension is 25 N. Find (a) the normal reaction, (b) the acceleration of the box. Take $g = 9.8$. Give answers to 3 s.f. if not exact.

$\sin\theta = \frac{5}{13}$, $\cos\theta = \frac{12}{13}$

(a) Vertical: $R + 25\sin\theta = 4g$, so $R = \frac{1923}{65}$

(b) Horizontal: $25\cos\theta - \mu R = 4a$

$\frac{300}{13} - 0.3\times\frac{1923}{65} = 4a$

$a = \frac{9231}{2600}$

Answer: (a) 29.6 N (b) 3.55 m s^{-2}

9 Separable differential equations (All boards)

Solve the differential equation

$y\frac{dy}{dx} = e^x$,

given that $y = 3$ when $x = 0$ $(y > 0)$. Give y in terms of x.

Separate: $\int y\,dy = \int e^x\,dx$, so $\frac{y^2}{2} = e^x + C$

$x = 0$, $y = 3$ gives $C = \frac{7}{2}$

$y^2 = 2e^x + 7$

Answer: $y = \sqrt{2e^x + 7}$

10 Normal approximation to the binomial (All boards)

$X \sim B(100, 0.6)$. Use a normal approximation to estimate $P(X \leq 58)$ to 3 significant figures.

$np = 60$, $npq = 24$, so $Y \sim N(60, 24)$

With continuity correction: $P(Y < 58.5)$

$z = \frac{58.5 - 60}{4.90} = -0.306$, $P(Z < -0.306) = 0.380$

Answer: 0.380