Day 323 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Normal distribution probabilities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X, the height (cm), is normally distributed with mean 169 and standard deviation 7.
Find (a) P(X > 180.2) (b) P(180.2 < X < 188.6), each to 3 s.f.

(a) $z = \frac{180.2 - 169}{7} = 1.60$, $P(Z > 1.60) = 0.0548$

(b) $z_1 = 1.60$, $z_2 = \frac{188.6 - 169}{7} = 2.80$

$P(180.2 < X < 188.6) = \Phi(2.80) - \Phi(1.60) = 0.0522$

Answer: (a) 0.0548 (b) 0.0522

2 Greatest and least values using R-form (All boards)

$g(x) = \frac{1}{18 + 8\sin x + 6\cos x}$

(a) Write 8 sin x + 6 cos x in the form R sin(x + α), with α acute.
(b) Find the greatest and least values of g(x) and the smallest positive x at which the greatest value occurs (1 d.p.).

$R = \sqrt{8^2 + 6^2} = 10$, $\alpha = \tan^{-1}\frac{6}{8} = 36.9°$

Denominator = 18 + 10 sin(x + 36.9°), ranging from 8 to 28

Greatest $g = \frac{1}{8}$ (denominator least), least $g = \frac{1}{28}$

Least denominator when sin(x + 36.9°) = −1: x + 36.9° = 270°
x = 233.1°

Answer: R = 10, α = 36.9°; greatest $\frac{1}{8}$, least $\frac{1}{28}$; x = 233.1°

3 Lift and cable tension (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A lift of mass 300 kg carries a person of mass 90 kg. The cable tension is 3705 N and the lift is accelerating. Find (a) the magnitude and direction of the acceleration, (b) the force of the floor on the person. Take g = 9.8.

Whole system (mass 390 kg), taking upwards as positive: T − 390 g = 390 a
3705 − 3822 = 390 a, so a = −0.3

(a) a is negative, so the acceleration is 0.3 m s^{-2} downwards
(b) Person: R − 90 g = 90 a, so R = 90(9.8 − 0.3)

Answer: (a) 0.3 m s^{-2} downwards (b) 855 N

4 Indices: solving equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $3^{x-3} = 9^{2x+3}$.

Write both sides as powers of 3.
$(3^1)^{x-3} = (3^2)^{2x+3}$
x − 3 = 2(2x + 3)
x − 3 = 4x + 6
−3x = 9

Answer: x = −3

5 Conditional probability and independence (All boards)

P(B) = 0.1, P(A) = 0.2 and P(A | B) = 0.15.
(a) Find P(A ∩ B) and P(A ∪ B).
(b) Find P(B | A).
(c) Determine whether A and B are independent.

(a) P(A ∩ B) = P(A | B)P(B) = 0.15 × 0.1 = 0.015
P(A ∪ B) = 0.2 + 0.1 − 0.015 = 0.285

(b) $P(B \mid A) = \frac{P(A \cap B)}{P(A)} = \frac{0.015}{0.2} = 0.075$

(c) P(A)P(B) = 0.2 × 0.1 = 0.02, which does not equal P(A ∩ B) = 0.015
So A and B are not independent

Answer: (a) 0.015, 0.285 (b) 0.075 (c) not independent

6 Collinear points and ratios (3D) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A, B and C have position vectors $\begin{pmatrix}-1\\4\\1\end{pmatrix}$, $\begin{pmatrix}-1\\6\\1\end{pmatrix}$ and $\begin{pmatrix}-1\\k\\1\end{pmatrix}$ relative to O, where k is a constant. Given that A, B and C are collinear, find k and the ratio AB : BC.

AB = $\begin{pmatrix}0\\2\\0\end{pmatrix}$, and AC = $\begin{pmatrix}0\\k-4\\0\end{pmatrix}$

AC = λAB: component 2 gives $-2 = 2\lambda$, so $\lambda = -1$

Hence $k = 4 + (-1) \times 2 = 2$

AC = −1 AB, so BC = −2 AB: AB : BC = 1 : 2

Answer: k = 2, AB : BC = 1 : 2

7 Rough incline: find the coefficient of friction (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle slides from rest down a rough plane inclined at α to the horizontal, where $\tan\alpha = \frac{4}{3}$. It moves 12.74 m in 2 s. Find (a) the coefficient of friction, (b) the speed after 2 s. Take g = 9.8.

$s = \frac{1}{2}at^2$: $12.74 = \frac{1}{2}\times a\times 2^2$, so a = 6.37

Along the slope: $mg\sin\alpha - \mu R = ma$, with $R = mg\cos\alpha$

$g(\sin\alpha - \mu\cos\alpha) = a$: $9.8(0.8 - 0.6\mu) = 6.37$

$0.8 - 0.6\mu = 0.65$, so $\mu = 0.25$

$v = at = 6.37\times 2$

Answer: $\mu = 0.25$; 12.74 m s^{-1}

8 Binomial expansion: finding n and k (All boards)

In the expansion of $(1 + kx)^n$, where n is a positive integer, the coefficient of x is 16 and the coefficient of x^2 is 112. Find n and k.

$nk = 16$ and $\frac{n(n-1)}{2}k^2 = 112$

$(n-1)k = \frac{2\times 112}{16} = 14$ (dividing the second equation by the first)

$nk - (n-1)k = k = 16 - 14 = 2$

$n = \frac{16}{2} = 8$

Answer: n = 8, k = 2

9 Partial fractions and binomial expansion (All boards)

$f(x) = \frac{19x-1}{(1-3x)(1+x)}$.

(a) Express f(x) in partial fractions.

(b) Hence expand f(x) up to x^2.

(c) State the range of x for which the expansion is valid.

(a) $f(x) = \frac{A}{(1-3x)} + \frac{B}{(1+x)}$: A = 4, B = −5

(b) $4(1-3x)^{-1} = 4(1 + 3x + 9x^2 \ldots)$

Add the series for both fractions.

(c) Need $|3x| < 1$ and $|x| < 1$

Answer: (a) $\frac{4}{(1-3x)} + \frac{-5}{(1+x)}$ (b) $-1 + 17x + 31x^2$ (c) $|x| < \frac{1}{3}$

10 Inverse trigonometric functions (AQA, Edexcel, OCR, MEI, Eduqas)

$f(x) = 2 + 2\arccos(2x)$.

Find the exact value of x for which $f(x) = \frac{4\pi + 6}{3}$.

Hence state the range of f.

$2\arccos(2x) = \frac{4\pi}{3}$, so $\arccos(2x) = \frac{2\pi}{3}$

$2x = \cos\frac{2\pi}{3} = -\frac{1}{2}$, so $x = -\frac{1}{4}$

arccos takes values from 0 to π, so f(x) runs from 2 to $2\pi + 2$

Answer: $x = -\frac{1}{4}$; range $2 \leq f(x) \leq 2\pi + 2$