Day 322 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Interpolation for median and percentiles (All boards)

The table summarises the journey time, t minutes of 53 items.
Use linear interpolation to estimate the lower quartile.

Position = 0.25 × 53 = 13.25
Cumulative frequency before the class 10 ≤ t < 20 is 12

Estimate = $10 + \frac{13.25 - 12}{3} \times 10$

= 14.2

Answer: 14.2

2 Completing the square (All boards)

$f(x) = x^2 - 10x + 17$.
(a) Write f(x) in the form $a(x + p)^2 + q$.
(b) State the minimum value of f(x) and the value of x at which it occurs.
(c) Solve f(x) = 0, giving surds in simplest form.

$f(x) = (x - 5)^2 - 8$
(b) minimum value -8 when $x = 5$
(c) $(x - 5)^2 = 8$, so $(x - 5)^2 = 8$
$x - 5 = \pm\sqrt{8}$
Answer: (a) $(x - 5)^2 - 8$ (b) minimum -8 at $x = 5$ (c) $x = 5 \pm 2\sqrt{2}$

3 Index equations solved with logarithms (All boards)

Solve $5^{3x + 3} = 2^{2x - 1}$.
Give your answer to 3 significant figures.

Take logs: $(3x + 3) \ln 5 = (2x - 1) \ln 2$
Collect x terms: $x(3 \ln 5 - 2 \ln 2) = -\ln 2 - 3 \ln 5$
$x = \frac{-\ln 2 - 3 \ln 5}{3 \ln 5 - 2 \ln 2}$
$x = -1.60$
Answer: $x = -1.60$

4 Speed-time graphs (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the speed-time graph for a train journey of 3570 m. Find (a) V, (b) the deceleration in the final stage.

(a) Distance = area of trapezium = ½(150 + 105) × V
3570 = 127.5 V, so V = 28
(b) Deceleration = V ÷ 25 = 28 ÷ 25
Answer: (a) V = 28 (b) 1.12 m s^{-2}

5 Three-set Venn diagram counts (All boards)

Of 33 students, 16 study Maths (M), 14 Physics (P) and 17 Chemistry (C).
6 study M and P, 6 study M and C, 8 study P and C, and 4 study all three.
One student is chosen at random. Given that a student studies exactly two subjects, find the probability that they study Physics.

Venn regions: M only 8, P only 4, C only 7,
MP only 2, MC only 2, PC only 4, all three 4, outside all 2
Exactly two subjects: 2 + 2 + 4 = 8
Of these, Physics: 2 + 4 = 6

Probability = $\frac{6}{8}$

Answer: $\frac{3}{4}$

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6 Addition formulae with given sin and cos (All boards)

A is acute and B is obtuse, with $\sin A = \frac{24}{25}$ and $\cos B = -\frac{5}{13}$.

Find the exact values of cos(A−B) and sin(A−B).

$\cos A = \frac{7}{25}$ (A acute), $\sin B = \frac{12}{13}$ (B obtuse), so $\tan A = \frac{24}{7}$, $\tan B = -\frac{12}{5}$

$\cos(A-B) = \cos A \cos B + \sin A \sin B = \frac{7}{25} \times (-\frac{5}{13}) + \frac{24}{25} \times \frac{12}{13} = \frac{253}{325}$

$\sin(A-B) = \sin A \cos B - \cos A \sin B = \frac{24}{25} \times (-\frac{5}{13}) - \frac{7}{25} \times \frac{12}{13} = -\frac{204}{325}$

Answer: $\cos(A-B) = \frac{253}{325}$, $\sin(A-B) = -\frac{204}{325}$

7 Periodic sequences (All boards)

A sequence is defined by $u_1 = \frac{3}{2}$ and $u_{n+1} = \frac{1}{1-u_n}$.

(a) Find u_2 and u_3.

(b) Find u_{100} and $\sum_{r=1}^{100} u_r$.

$u_2 = -2$, $u_3 = \frac{1}{3}$, $u_4 = \frac{3}{2}$

$u_4 = u_1$, so the sequence repeats with period 3.

$100 = 3 \times 33 + 1$, so $u_{100} = u_1 = \frac{3}{2}$

Sum of one period $= -\frac{1}{6}$; 33 full periods plus 1 extra term(s)

Answer: (a) $-2, \frac{1}{3}$ (b) $u_{100} = \frac{3}{2}$, sum $= -4$

8 Equilibrium on a slope: range of force (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A 2 kg particle rests on a rough plane inclined at α, $\tan\alpha = \frac{4}{3}$, $\mu = 0.5$. A force of P N acts up a line of greatest slope and the particle stays at rest. Find the range of values of P. Take g = 9.8.

$\sin\alpha = 0.8$, $\cos\alpha = 0.6$, $R = 2g\cos\alpha = 11.76$ N

Limiting friction $= \mu R = 5.88$ N

P smallest: friction acts up the slope: $P + \mu R = 2g\sin\alpha$, $P = 15.68 - 5.88$

P greatest: friction acts down the slope: $P = 15.68 + 5.88$

Answer: $9.8 \text{ N} \leq P \leq 21.56 \text{ N}$

9 Inverse trigonometric functions (AQA, Edexcel, OCR, MEI, Eduqas)

Find the exact value of $\sin(\arcsin\frac{3}{5} + \arcsin\frac{5}{13})$.

Let $A = \arcsin\frac{3}{5}$ and $B = \arcsin\frac{5}{13}$, both acute

$\cos A = \frac{4}{5}$, $\cos B = \frac{12}{13}$

$\sin(A+B) = \sin A \cos B + \cos A \sin B = \frac{3}{5} \times \frac{12}{13} + \frac{4}{5} \times \frac{5}{13} = \frac{56}{65}$

Answer: $\frac{56}{65}$

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10 Surds: equation with surd coefficients (AQA, Edexcel, OCR, MEI, Eduqas)

Solve $x(\sqrt{5} - 2) = 4 + 2\sqrt{5}$, writing x in the form $p + q\sqrt{5}$.

$x = \dfrac{4 + 2\sqrt{5}}{\sqrt{5} - 2}$

Multiply top and bottom by $\sqrt{5} + 2$.

Numerator: $(4 + 2\sqrt{5})(\sqrt{5} + 2) = 18 + 8\sqrt{5}$

Denominator: $5 - 4 = 1$

Answer: $x = 18 + 8\sqrt{5}$