Day 321 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 R-form and solving a sin x + b cos x = c (All boards)

(a) Express $8\sin\theta - 15\cos\theta$ in the form $R\sin(\theta - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. Give α to 1 d.p.
(b) Solve $8\sin\theta - 15\cos\theta = 1$ for $0° \leq \theta < 360°$, giving answers to 1 d.p.

$R^2 = 8^2 + 15^2 = 289$, so $R = 17$; $\tan\alpha = \frac{15}{8}$ gives $\alpha = 61.9°$

$17\sin(\theta - 61.9°) = 1$, so $\sin(\theta - 61.9°) = \frac{1}{17}$

Principal value 3.4°, then use the symmetry of the sine graph
$\theta = 65.3°, 238.6°$

Answer: (a) R = 17, α = 61.9° (b) θ = 65.3°, 238.6°

2 Constant acceleration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A cyclist passes P with speed 6 m s^{-1} and accelerates uniformly. She travels 60.8 m in the next 8 s, reaching Q. Find (a) her acceleration, (b) her speed at Q, (c) her speed at the midpoint of PQ. Give answers to 3 s.f. if not exact.

(a) $s = ut + \frac{1}{2}at^2$: $60.8 = 6\times8 + \frac{1}{2}\times a\times8^2$, so $a = 0.4$
(b) $v = u + at = 6 + 0.4\times8 = 9.2$
(c) For the midpoint, $s = 30.4$: $v^2 = u^2 + 2as = 6^2 + 2\times0.4\times30.4$
$v^2 = 60.32$

Answer: (a) 0.4 m s^{-2} (b) 9.2 m s^{-1} (c) 7.77 m s^{-1}

3 Polynomial division (All boards)

Find the quotient and the remainder when $9x^3 + 3x^2 + 9x + 6$ is divided by $(3x + 1)$.

Quotient (matching leading terms in turn): $3x^2 + 3$
$(3x + 1)(3x^2 + 3) = 9x^3 + 3x^2 + 9x + 3$
Remainder $= 9x^3 + 3x^2 + 9x + 6 - (9x^3 + 3x^2 + 9x + 3) = 3$

Answer: quotient $3x^2 + 3$, remainder 3

4 Newton's second law with vectors (AQA, Edexcel, OCR, MEI, Eduqas)

A particle of mass 3 kg is acted on by forces $\mathbf{F_1} = (33\mathbf{i} + 19\mathbf{j})$ N and $\mathbf{F_2}$. Its acceleration is $(3\mathbf{i} + 4\mathbf{j})$ m s^{-2}. Find (a) $\mathbf{F_2}$, (b) the angle between $\mathbf{F_2}$ and $\mathbf{i}$, to 1 d.p.

$\mathbf{F} = m\mathbf{a}$: $\mathbf{F_1} + \mathbf{F_2} = 3(3\mathbf{i} + 4\mathbf{j}) = 9\mathbf{i} + 12\mathbf{j}$
(a) $\mathbf{F_2} = (9\mathbf{i} + 12\mathbf{j}) - (33\mathbf{i} + 19\mathbf{j}) = -24\mathbf{i} - 7\mathbf{j}$
(b) $|\mathbf{F_2}| = \sqrt{(-24^2 + -7^2)} = 25$
$\cos\theta = -24 \div 25$, so $\theta = 163.7°$

Answer: (a) $\mathbf{F_2} = (-24\mathbf{i} - 7\mathbf{j})$ N (b) 163.7°

5 Sum of multiples in a range (All boards)

Find the sum of all the multiples of 8 that lie strictly between 85 and 579.

First multiple 88, last multiple 576, common difference 8.

Number of terms $n = \frac{576 - 88}{8} + 1 = 62$

$S = \frac{62}{2}(88 + 576) = 20584$

Answer: 20584

6 Exponential growth and decay modelling (All boards)

A car costs £15000 new. Its value falls by 10% each year, so after t years its value is £V.
(a) Write down V in terms of t. (b) Find, to 3 s.f., the time when the value is £5000.

$V = 15000 \times 0.9^t$

$15000 \times 0.9^t = 5000$ gives $0.9^t = \frac{1}{3}$

$t = \frac{\ln(1}{3)/\ln 0.9} = 10.4$ years

Answer: (a) $V = 15000 \times 0.9^t$ (b) t = 10.4 years

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7 Combining two data sets (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The times in seconds of two groups: group X has n = 27, mean 42, sd 7;
group Y has n = 24, mean 25, sd 7.
Find the mean and standard deviation of all 51 values (3 s.f. for the sd).

$\Sigma x = 27 \times 42 + 24 \times 25 = 1734$, mean = 34

Σx^2 (A) $= 27(7^2 + 42^2) = 48951$

Σx^2 (B) $= 24(7^2 + 25^2) = 16176$

Variance $= \frac{65127}{51} - 34^2 = 121$, so sd = 11.0

Answer: mean 34, sd 11.0

8 Differential equations in context (All boards)

A drink at 102°C is left in a room at 22°C. Its temperature θ °C satisfies $\frac{d\theta}{dt} = -k(\theta - 22)$.
After 5 minutes the temperature is 67°C. Find k, then the time for it to reach 32°C, to 3 s.f.

$\int \frac{1}{\theta - 22}\, d\theta = \int -k\, dt$, so $\ln(\theta - 22) = -kt + C$

t = 0: $\theta - 22 = 80$, so $\theta - 22 = 80e^{-kt}$

t = 5: $45 = 80e^{-5k}$, so $k = \frac{\ln(\frac{16}{9})}{5} = 0.115$

$10 = 80e^{-kt}$: $t = \frac{\ln(8)}{k} = 18.1$ minutes

Answer: k = 0.115, t = 18.1 minutes

9 Modelling with a normal distribution (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

At one location, the daily mean cloud cover in a month is modelled by $N(5.6, 1.7^2)$.
(a) Find the probability that on a randomly chosen day the value exceeds 7 oktas.
(b) Estimate the number of such days in a period of 92 days.

(a) $z = \frac{7 - 5.6}{1.7} = 0.824$

$P(Z > 0.824) = 0.205$

(b) $92 \times 0.205 = 18.9 \approx 19$

Answer: (a) 0.205 (b) 19 days

10 Circle through two points with centre on a line (AQA, Edexcel, OCR, MEI, Eduqas)

A circle passes through A(5, 7) and B(−9, 5).
Its centre lies on the line y = 3x + 2.
Find the equation of the circle.

Centre (h, 3h + 2) is equidistant from A and B.

$(h - 5)^2 + (3h - 5)^2 = (h + 9)^2 + (3h - 3)^2$

Solving gives h = −1, so centre (−1, −1)

$r^2 = (6)^2 + (8)^2 = 100$

Answer: $(x + 1)^2 + (y + 1)^2 = 100$