Day 320 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Combined inequalities (All boards)

(a) Solve $2x + 5 \geq -1$.

(b) Solve $x^2 + 4x - 5 \geq 0$.

(c) Hence find the values of x satisfying both inequalities.

(a) $2x + 5 \geq -1$ so $x \geq -3$

(b) $(x + 5)(x - 1) \geq 0$: critical values -5 and 1

(b) gives $x \leq -5$ or $x \geq 1$

(c) Overlap of the two sets.

Answer: (a) $x \geq -3$ (b) $x \leq -5$ or $x \geq 1$ (c) $x \geq 1$

2 Chain rule and gradients (All boards)

$y = \sqrt{3x^2 - 8}$.

Find $\frac{dy}{dx}$ and the gradient of the curve at $x = 2$.

$y = (3x^2 - 8)^{\frac{1}{2}}$, so $\frac{dy}{dx} = \frac{1}{2}(3x^2 - 8)^{-\frac{1}{2}} \times 6x = \frac{3x}{\sqrt{3x^2 - 8}}$

At $x = 2$: $\sqrt{3x^2 - 8} = 2$, so gradient $= \frac{6}{2} = 3$

Answer: $\frac{dy}{dx} = \frac{3x}{\sqrt{3x^2 - 8}}$; gradient = 3

3 Completing the square (All boards)

$f(x) = -x^2 - 10x - 22$.

(a) Write f(x) in the form $a(x + p)^2 + q$.

(b) State the maximum value of f(x) and the value of x at which it occurs.

(c) Solve $f(x) = 0$, giving surds in simplest form.

$f(x) = -(x + 5)^2 + 3$

(b) maximum value 3 when $x = -5$

(c) $-(x + 5)^2 = -3$, so $(x + 5)^2 = 3$

$x + 5 = \pm\sqrt{3}$

Answer: (a) $-(x + 5)^2 + 3$ (b) maximum 3 at $x = -5$ (c) $x = -5 \pm \sqrt{3}$

4 Geometric distribution (CIE)

On each independent trial, the probability that a basketball player scores is $\frac{2}{5}$.

X is the number of trials up to and including the first success.

(a) Find P(X = 6) and E(X).

(b) Find the least m with $P(X \leq m) > 0.99$.

(a) $P(X = 6) = (\frac{3}{5})^5 \times \frac{2}{5} = \frac{486}{15625}$; $E(X) = \frac{1}{p} = 2$

(b) $P(X \leq m) = 1 - (\frac{3}{5})^m > 0.99$

$(\frac{3}{5})^m < 0.01$, so $m > 9.02$

Least integer $m = 10$

Answer: (a) $\frac{486}{15625}$, E(X) = 2 (b) m = 10

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5 Small angle approximations: solving an equation (AQA, Edexcel, OCR, MEI, Eduqas)

θ is small and measured in radians. Use small angle approximations to show that
$5\sin\theta + \cos 2\theta = 2$
can be written as $a\theta^2 + b\theta + c = 0$ with integers a, b, c. Hence find the approximate solution with $0 < \theta < 0.8$, to 3 s.f.

$\sin\theta \approx \theta$, $\cos 2\theta \approx 1 - \frac{4}{2}\theta^2$

$5 \times \theta + 1(1 - \frac{4}{2}\theta^2) = 2$

Multiply by 2 and rearrange: $4\theta^2 - 10\theta + 2 = 0$

Divide through by 2: $2\theta^2 - 5\theta + 1 = 0$

$\theta = \frac{5 \pm \sqrt{17}}{4}$, so $\theta = 0.219$

Answer: a = 2, b = −5, c = 1; $\theta \approx 0.219$

6 Inverse of a quadratic function (All boards)

$f(x) = 3x^2 + 4$, $x \geq 0$.
Find $f^{-1}(x)$ and state its domain.

$f(x) = 3x^2 + 4$

Let $y = f(x)$: $x^2 = \frac{y - 4}{3}$

$x \geq 0$, so take the positive root: $x = \sqrt{\frac{y - 4}{3}}$

Answer: $f^{-1}(x) = \sqrt{\frac{x - 4}{3}}$, $x \geq 4$

7 Equations with cosec x and cot x (All boards)

Solve $4\text{cosec}^2 x - 20\cot x + 20 = 0$
for $0° < x < 360°$. Give answers to 1 d.p.

Use $\text{cosec}^2 x \equiv 1 + \cot^2 x$: $4\cot^2 x - 20\cot x + 24 = 0$
$\cot x = 3$ or $\cot x = 2$

$\cot x = 3$ means $\tan x = \frac{1}{3}$: $x = 18.4°, 198.4°$

$\cot x = 2$ means $\tan x = \frac{1}{2}$: $x = 26.6°, 206.6°$

Answer: x = 18.4°, 26.6°, 198.4°, 206.6°

8 Projectile from a cliff top (AQA, Edexcel, OCR, MEI, Eduqas)

A stone is projected from a cliff top 32.4 m above the sea at 40 m s^{-1}, at angle α above the horizontal, $\tan\alpha = \frac{3}{4}$. Find (a) the time to reach the sea, (b) the horizontal distance travelled, (c) the speed on impact. Take g = 9.8. Give answers to 3 s.f. if not exact.

Horizontal component 32, vertical component 24 (upwards)
(a) $-32.4 = 24\,t - 4.9t^2$, so $4.9t^2 - 24\,t - 32.4 = 0$
$t = [24 + \sqrt{1211.04}] \div 9.8 = 6$ (taking the positive root)
(b) Distance = 32×6
(c) Vertical speed = 24 − 9.8×6 = −34.8; speed = $\sqrt{(32^2 + 34.8^2)}$

Answer: (a) 6 s (b) 192 m (c) 47.3 m s^{-1}

9 Projectile: perpendicular directions (AQA, Edexcel, OCR, MEI, Eduqas)

A ball is projected from the ground at 11 m s^{-1} at angle α above the horizontal, $\tan\alpha = \frac{15}{8}$. At time T s, while in flight, it is moving perpendicular to its initial direction. Find (a) T, (b) the height of the ball then. Take g = 9.8. Give answers to 3 s.f. if not exact.

Initial velocity $(\frac{88}{17}, \frac{165}{17})$. At time T the velocity is $(\frac{88}{17}, \frac{165}{17} - 9.8T)$.

Perpendicular: scalar product = 0: $(\frac{88}{17})^2 + \frac{165}{17}(\frac{165}{17} - 9.8T) = 0$

$11^2 - 9.8T \times \frac{165}{17} = 0$ (using $\sin^2\alpha + \cos^2\alpha = 1$)

(a) $T = 11 \div (9.8 \times \frac{15}{17}) = \frac{187}{147}$

(b) $h = \frac{165}{17} \times \frac{187}{147} - 4.9 \times \frac{187}{147}^2 = \frac{2783}{630}$

Answer: (a) T = 1.27 s (b) 4.42 m

10 Three-set Venn diagram counts (All boards)

Of 40 students, 16 study Maths (M), 17 Physics (P) and 19 Chemistry (C).
7 study M and P, 7 study M and C, 6 study P and C, and 4 study all three.
One student is chosen at random. Given that a student does not study Maths, find the probability that they study Physics.

Venn regions: M only 6, P only 8, C only 10,
MP only 3, MC only 3, PC only 2, all three 4, outside all 4
Not Maths: 40 − 16 = 24
Physics but not Maths: 17 − 7 = 10

Probability = $\frac{10}{24}$

Answer: $\frac{5}{12}$