Day 319 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Arithmetic sequences: finding a and d (All boards)

The 2nd term of an arithmetic sequence is -10 and the 9th term is -31.
Find the first term and common difference, and the sum of the first 15 terms.

$u_9 - u_2 = 7\,d = -31 - (-10)$ so $d = -3$

$a + 1\,d = -10$ so $a = -7$

$S_{15} = \frac{15}{2}(2 \times (-7) + 14 \times (-3))$

Answer: $a = -7$, $d = -3$, $S_{15} = -420$

2 Comparing data sets from the large data set (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Summary statistics for the daily total rainfall (mm) in the same month are:
Heathrow: mean 14.5, standard deviation 1.2;
Leuchars: mean 28.5, standard deviation 1.9.
Make two comparisons.

Means: Leuchars has the higher mean (28.5 against 14.5), so on average the total rainfall is greater at Leuchars
Standard deviations: Leuchars is larger at Leuchars (1.9 against 1.2)
so the readings at Leuchars are more variable and those at Heathrow are more consistent
Answer: Leuchars higher on average; Leuchars more variable

3 Constant acceleration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A cyclist passes P with speed 2 m s^{-1} and accelerates uniformly. She travels 45 m in the next 10 s, reaching Q. Find (a) her acceleration, (b) her speed at Q, (c) her speed at the midpoint of PQ. Give answers to 3 s.f. if not exact.

(a) $s = ut + \frac{1}{2}at^2$: $45 = 2\times10 + \frac{1}{2}\times a\times10^2$, so $a = 0.5$
(b) $v = u + at = 2 + 0.5\times10 = 7$
(c) For the midpoint, $s = 22.5$: $v^2 = u^2 + 2as = 2^2 + 2\times0.5\times22.5$
$v^2 = 26.5$
Answer: (a) 0.5 m s^{-2} (b) 7 m s^{-1} (c) 5.15 m s^{-1}

4 Periodic sequences (All boards)

A sequence is defined by $u_1 = 5$ and $u_{n+1} = \frac{u_n + 1}{u_n - 1}$.

(a) Find u_2 and u_3.

(b) Find u_{101} and $\sum_{r=1}^{101} u_r$.

$u_2 = \frac{3}{2}$, $u_3 = 5$, $u_4 = \frac{3}{2}$

$u_3 = u_1$, so the sequence repeats with period 2.
$101 = 2 \times 50 + 1$, so $u_{101} = u_1 = 5$

Sum of one period = $\frac{13}{2}$; 50 full periods plus 1 extra term(s)

Answer: (a) $\frac{3}{2}$, 5 (b) $u_{101} = 5$, sum = 330

5 Implicit differentiation: tangent to a curve (All boards)

A curve has equation $x^2 - 4xy + 2x - y = 14$.
Show that P$(-3, 1)$ lies on the curve and find the equation of the tangent at P in the form $ax + by + c = 0$ with integer a, b, c.

Differentiate (product rule on the xy term): $(2x - 4y + 2) + (-4x - 1)\frac{dy}{dx} = 0$

At P$(-3, 1)$: $-8 + (11)\frac{dy}{dx} = 0$, so $\frac{dy}{dx} = \frac{8}{11}$

Tangent: $y - 1 = \frac{8}{11}(x + 3)$, i.e. $8x - 11y + 35 = 0$

Answer: $8x - 11y + 35 = 0$

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6 Coding mean and standard deviation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

For 40 values of x, the coded variable $y = \frac{x - 30}{4}$ gives

$\Sigma y = 80$ and $\Sigma y^2 = 320$.

Find the mean and standard deviation of x.

$\bar{y} = \frac{80}{40} = 2$

$\sigma_y^2 = \frac{320}{40} - 2^2 = 4$, so $\sigma_y = 2$

$x = 4y + 30$, so mean $= 4 \times 2 + 30 = 38$

Standard deviation $= 4 \times 2 = 8$

Answer: mean 38, sd 8

7 Reducing a relationship to a straight line (graph) (All boards)

The diagram shows the graph of ln y against x for $y = A b^x$.

Find the exact values of A and b.

$\ln y = \ln A + x \ln b$, a straight line with gradient $\ln b$ and intercept $\ln A$

Intercept $\ln A = 3$, so $A = e^3$

Gradient $\ln b = \frac{6 - 3}{2 - 0} = \frac{3}{2}$, so $b = e^{\frac{3}{2}}$

Answer: $A = e^3$, $b = e^{\frac{3}{2}}$

8 Circle touching both axes (AQA, Edexcel, OCR, MEI, Eduqas)

A circle touches both coordinate axes and passes through the point $(4, -2)$. Its centre lies in the fourth quadrant.

Find the equations of the two possible circles.

Centre $(r, -r)$, radius $r > 0$.

Through $(4, -2)$: $(4 - r)^2 + (-2 + r)^2 = r^2$

$r^2 - 12r + 20 = 0$

$(r - 2)(r - 10) = 0$, so $r = 2$ or 10

Answer: $(x - 2)^2 + (y + 2)^2 = 4$ or $(x - 10)^2 + (y + 10)^2 = 100$

9 Integration by parts (All boards)

Use integration by parts to find the exact value of

$\int_1^e x^2 \ln x \, dx$.

$u = \ln x$, $\frac{dv}{dx} = x^2$: $\int x^2 \ln x \, dx = \left[\frac{x^3 \ln x}{3}\right] - \int \frac{x^2}{3} \, dx$

$= \left[\frac{x^3 \ln x}{3} - \frac{x^3}{9}\right]$

$= \frac{2e^3 + 1}{9}$

Answer: $\frac{2e^3 + 1}{9}$

10 Connected particles (rough table, string slack) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Particle A (4 kg) lies on a rough table, $\mu = 0.25$, joined by a light string over a smooth pulley to particle B (6 kg), 0.8 m above the floor. Released from rest; B lands without rebounding. Find (a) the acceleration of A before B lands, (b) the further distance A moves. Take $g = 9.8$.

B: $6g - T = 6a$; A: $T - 0.25 \times 4g = 4a$

Adding: $(6 - 0.25 \times 4)g = 10a$, so $a = 4.9$

Speed of B and A as B lands: $v^2 = 2 \times 4.9 \times 0.8 = 7.84$

After B lands, A decelerates at $\mu g = 0.25 \times 9.8 = 2.45$

$0 = v^2 - 2(\mu g)s$, $s = 7.84 \div 4.9$

Answer: (a) 4.9 m s^{-2} (b) 1.6 m