Day 318 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Index equations solved with logarithms (All boards)

Solve $5^{x+1} = 3^{x+2}$.
Give your answer to 3 significant figures.

Take logs: $(x + 1) \ln 5 = (x + 2) \ln 3$
Collect x terms: $x(\ln 5 - \ln 3) = 2 \ln 3 - \ln 5$
$x = \dfrac{2 \ln 3 - \ln 5}{\ln 5 - \ln 3}$
$x = 1.15$
Answer: $x = 1.15$

2 Parametric equations: Cartesian form (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has parametric equations $x = t + 3$, $y = t^2$.
Find a Cartesian equation of the curve.

From x: $t = x - 3$
$y = (x - 3)^2$
Expand.
Answer: $y = x^2 - 6x + 9$

3 Speed-time graphs (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the speed-time graph for a train journey of 1820 m. Find (a) V, (b) the deceleration in the final stage.

(a) Distance = area of trapezium = ½(90 + 40) × V
1820 = 65 V, so V = 28
(b) Deceleration = V ÷ 25 = 28 ÷ 25
Answer: (a) V = 28 (b) 1.12 m s^{-2}

4 Circle: centre, radius and position of a point (AQA, Edexcel, OCR, MEI, Eduqas)

A circle has equation $x^2 + y^2 - 8x + 6y + 15 = 0$.
(a) Find the centre and radius of the circle.
(b) Determine whether P(6, −3) lies inside or outside the circle.

Complete the square: $(x - 4)^2 + (y + 3)^2 = 10$
Centre (4, −3), radius $\sqrt{10}$
Distance^2 from centre to P = $2^2 + 0^2 = 4$
Compare with $r^2 = 10$
Answer: (a) centre (4, −3), radius $\sqrt{10}$ (b) P is inside the circle

5 Scalar product, magnitude of a sum (CIE (9709); Further Maths on other boards)

$|a| = 6$, $|b| = 4$ and the angle between a and b is 60°.
Find the exact values of $a \cdot b$ and $|a + b|$.

$a \cdot b = |a||b|\cos 60° = 6 \times 4 \times \dfrac{1}{2} = 12$
$|a + b|^2 = |a|^2 + 2a \cdot b + |b|^2 = 36 + 24 + 16 = 76$
$|a + b| = 2\sqrt{19}$
Answer: $a \cdot b = 12$, $|a + b| = 2\sqrt{19}$

6 Finding an unknown standard deviation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X is normally distributed with mean μ and standard deviation 12.
Given that P(X > 77) = 0.3085, find μ to 3 significant figures.

P(Z > z) = 0.3085, so z = 0.500
$\dfrac{77 - \mu}{12} = 0.500$
$\mu = 77 - (0.500)(12) = 71.0$
Answer: $\mu = 71.0$

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7 Connected rates of change: spheres, circles and cubes (AQA, Edexcel, OCR, MEI, Eduqas)

A circular ripple has area A cm^2 increasing at a constant 3 cm^2s^{-1}.
Find the rate at which the circumference is increasing when the radius is 2 cm.

$A = \pi r^2$, so $\frac{dA}{dr} = 2\pi r$ and $\frac{dr}{dt} = \frac{3}{2\pi r}$

$C = 2\pi r$, so $\frac{dC}{dt} = 2\pi \times \frac{dr}{dt} = 2\pi \times \frac{3}{2\pi \times 2} = \frac{3}{2}$

Answer: $\frac{3}{2}$ cm/s

8 Three-set Venn diagram counts (All boards)

Of 36 students, 19 study Maths (M), 22 Physics (P) and 15 Chemistry (C).
10 study M and P, 8 study M and C, 9 study P and C, and 4 study all three.
One student is chosen at random. Given that a student studies Chemistry, find the probability that they study all three.

Venn regions: M only 5, P only 7, C only 2,
MP only 6, MC only 4, PC only 5, all three 4, outside all 3
Studies Chemistry: 15
Studies all three: 4

Probability = $\frac{4}{15}$

Answer: $\frac{4}{15}$

9 Periodic sequences (All boards)

A sequence is defined by $u_1 = 3$ and $u_{n+1} = \frac{1}{1 - u_n}$.

(a) Find u_2 and u_3.

(b) Find u_{50} and $\sum_{r=1}^{50} u_r$.

$u_2 = -\frac{1}{2}$, $u_3 = \frac{2}{3}$, $u_4 = 3$

$u_4 = u_1$, so the sequence repeats with period 3.

$50 = 3 \times 16 + 2$, so $u_{50} = u_2 = -\frac{1}{2}$

Sum of one period = $\frac{19}{6}$; 16 full periods plus 2 extra term(s)

Answer: (a) $-\frac{1}{2}$, $\frac{2}{3}$ (b) $u_{50} = -\frac{1}{2}$, sum = $\frac{319}{6}$

10 Particle supported by two strings (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A 3 kg particle hangs in equilibrium from two light strings, as shown. Find the tension in each string (T_1 in the string at angle α). Take g = 9.8.

$\sin\alpha = 0.8$, $\cos\alpha = 0.6$, $\sin\beta = 0.6$, $\cos\beta = 0.8$
Horizontal: $T_1 \cos\alpha = T_2 \cos\beta$, so $T_2 = 0.75\, T_1$
Vertical: $T_1 \sin\alpha + T_2 \sin\beta = 3\,g = 29.4$
$T_1(0.8 + 0.45) = 29.4$
$T_1 = 23.5$, $T_2 = 17.6$
Answer: $T_1 = 23.52$ N, $T_2 = 17.64$ N