Day 317 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Completing the square (All boards)

$f(x) = -2x^2 - 20x - 45$.

(a) Write f(x) in the form $a(x + p)^2 + q$.

(b) State the maximum value of f(x) and the value of x at which it occurs.

(c) Solve f(x) = 0, giving surds in simplest form.

$f(x) = -2(x + 5)^2 + 5$

(b) maximum value 5 when $x = -5$

(c) $-2(x + 5)^2 = -5$, so $(x + 5)^2 = \frac{5}{2}$

$x + 5 = \pm\sqrt{\frac{5}{2}}$

Answer: (a) $-2(x + 5)^2 + 5$ (b) maximum 5 at $x = -5$ (c) $x = -5 \pm \frac{\sqrt{10}}{2}$

2 Newton-Raphson method (AQA, Edexcel, OCR, MEI, Eduqas)

$f(x) = x^2 - 7$. Show that the Newton-Raphson formula gives $x_{n+1} = \frac{1}{2}(x_n + \frac{7}{x_n})$, and use it with $x_0 = \frac{3}{2}$ to find x_2 as an approximation to $\sqrt{7}$, to 4 d.p.

$f'(x) = 2x$, so $x_{n+1} = x_n - \frac{x_n^2 - 7}{2x_n} = \frac{1}{2}(x_n + \frac{7}{x_n})$

$x_1 = \frac{1}{2}(\frac{3}{2} + \frac{7}{\frac{3}{2}}) = \frac{37}{12}$

$x_2 = 2.6768$

Answer: $x_2 = 2.6768$

3 Venn diagrams and conditional probability (All boards)

P(A) = 0.65, P(B) = 0.75 and $P(A \cap B) = 0.5$.

Find

(a) $P(A \cup B)$

(b) P(A | B)

(c) $P(A' \cap B')$

(a) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.65 + 0.75 - 0.5 = 0.9$

(b) $P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{0.5}{0.75} = \frac{2}{3}$

(c) $P(A' \cap B') = 1 - P(A \cup B) = 1 - 0.9 = 0.1$

Answer: (a) 0.9 (b) $\frac{2}{3}$ (c) 0.1

4 Algebraic fractions (All boards)

Simplify fully $\frac{x^2 + 2x - 3}{3x^2 - 8x + 5}$.

Factorise the numerator: $(x - 1)(x + 3)$

Factorise the denominator: $(x - 1)(3x - 5)$

Cancel the common factor $(x - 1)$.

Answer: $\frac{x + 3}{3x - 5}$

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5 Projectile from and to the ground (AQA, Edexcel, OCR, MEI, Eduqas)

A ball is projected from a point O on horizontal ground at 22 m s^{-1} at angle α above the horizontal, $\tan\alpha = \frac{12}{5}$, and lands at A. Find (a) the time of flight, (b) the greatest height, (c) OA. Take g = 9.8. Give answers to 3 s.f. if not exact.

Horizontal component = $22\times\frac{5}{13} = \frac{110}{13}$; vertical component = $22\times\frac{12}{13} = \frac{264}{13}$

(a) Vertical: $0 = \frac{264}{13}t - 4.9t^2$, so $t = \frac{2640}{637}$

(b) At the top: $0 = \left(\frac{264}{13}\right)^2 - 2\times9.8\times H$, so $H = \frac{174240}{8281}$

(c) OA = $\frac{110}{13}\times\frac{2640}{637}$

Answer: (a) 4.14 s (b) 21.0 m (c) 35.1 m

6 Rationalising the denominator (AQA, Edexcel, OCR, MEI, Eduqas)

Express $\frac{2-2\sqrt{7}}{6+2\sqrt{7}}$ in the form $p + q\sqrt{7}$, where p and q are rational numbers.

Multiply top and bottom by $6 - 2\sqrt{7}$.

Denominator: $(6 + 2\sqrt{7})(6 - 2\sqrt{7}) = 8$

Numerator: $(2 - 2\sqrt{7})(6 - 2\sqrt{7}) = 40 - 16\sqrt{7}$

So the expression is $\frac{40-16\sqrt{7}}{8}$.

Answer: $5 - 2\sqrt{7}$

7 Continuous random variable: finding k and E(X) (CIE)

The random variable X has probability density function $f(x) = kx^3$ for $0 \le x \le 6$, and 0 otherwise.
(a) Find k. (b) Find E(X). (c) Find P(X > 4).

(a) $\int_0^6 f(x)\,dx = 1$ gives $324k = 1$, so $k = \frac{1}{324}$

(b) $E(X) = \int x\,f(x)\,dx = \frac{24}{5}$

(c) $P(X > 4) = \int_4^6 f(x)\,dx = \frac{65}{81}$

Answer: (a) $k = \frac{1}{324}$ (b) $\frac{24}{5}$ (c) $\frac{65}{81}$

8 Parametric differentiation: tangent and normal (All boards)

A curve has parametric equations $x = t + \frac{1}{t}$, $y = t - \frac{1}{t}$.

Find $\frac{dy}{dx}$ in terms of t, and the equation of the normal at the point where t = 2, in the form ax + by + c = 0 (integers).

$\frac{dy}{dx} = \frac{dy}{dt \div dx/dt} = \frac{t^2+1}{t^2-1}$

t = 2: point $(\frac{5}{2}, \frac{3}{2})$, gradient $\frac{5}{3}$

Normal gradient = $-1 \div \frac{5}{3} = -\frac{3}{5}$, so normal: $3x + 5y - 15 = 0$

Answer: $\frac{dy}{dx} = \frac{t^2+1}{t^2-1}$; $3x + 5y - 15 = 0$

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9 Projectile from a cliff top (AQA, Edexcel, OCR, MEI, Eduqas)

A stone is projected from a cliff top 26.1 m above the sea at 10 m s^{-1}, at angle α above the horizontal, $\tan\alpha = \frac{3}{4}$. Find (a) the time to reach the sea, (b) the horizontal distance travelled, (c) the speed on impact. Take g = 9.8. Give answers to 3 s.f. if not exact.

Horizontal component 8, vertical component 6 (upwards)
(a) $-26.1 = 6\,t - 4.9t^2$, so $4.9t^2 - 6\,t - 26.1 = 0$
t = [6 + √547.56] ÷ 9.8 = 3 (taking the positive root)
(b) Distance = 8×3
(c) Vertical speed = $6 - 9.8\times3 = -23.4$; speed = $\sqrt{(8^2 + 23.4^2)}$
Answer: (a) 3 s (b) 24 m (c) 24.7 m s^{-1}

10 Foot of perpendicular and distance to a line (CIE (9709); Further Maths on other boards)

The line l has equation $\mathbf{r} = \begin{pmatrix}1\\3\\-1\end{pmatrix} + t\begin{pmatrix}-2\\-2\\0\end{pmatrix}$.

Find the coordinates of the foot of the perpendicular from P(−2, 2, −1) to l, and the exact distance from P to l.

Foot F = A + t d: $\mathrm{PF} = \begin{pmatrix}-2t+3\\-2t+1\\0\end{pmatrix}$
PF·d = 0 gives $8t - 8 = 0$, so t = 1
F = (−1, 1, −1); $\mathrm{PF} = \begin{pmatrix}1\\-1\\0\end{pmatrix}$, so $|\mathrm{PF}| = \sqrt{2}$
Answer: F(−1, 1, −1), distance $\sqrt{2}$