Day 316 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Lift and cable tension (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A lift of mass 300 kg carries a person of mass 75 kg. The cable tension is 3487.5 N and the lift is accelerating. Find (a) the magnitude and direction of the acceleration, (b) the force of the floor on the person. Take g = 9.8.

Whole system (mass 375 kg), taking upwards as positive: $T - 375\,g = 375\,a$
$3487.5 - 3675 = 375\,a$, so $a = -0.5$

(a) a is negative, so the acceleration is 0.5 m s^{-2} downwards
(b) Person: $R - 75\,g = 75\,a$, so $R = 75(9.8 - 0.5)$

Answer: (a) 0.5 m s^{-2} downwards (b) 697.5 N

2 Inverse normal (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X, the length (mm), is N(82, 2^2). The lowest 15% of values are below k.
Find k to 3 significant figures.

$P(X < k) = 0.15$
$z = \Phi^{-1}(0.15) = -1.04$
$k = 82 + (-1.04) \times 2 = 79.9$

Answer: 79.9

3 Friction: find μ then stopping distance (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A 4 kg box is pulled from rest along rough horizontal ground by a horizontal force of 22.8 N and moves 10 m in 5 s. Find (a) the coefficient of friction. The force is then removed; (b) find the further distance moved. Take g = 9.8. Give answers to 3 s.f. if not exact.

$s = \frac{1}{2}at^2$: $10 = \frac{1}{2}a \times 5^2$, so $a = 0.8$
(a) $P - \mu 4\,g = 4\,a$: $22.8 - 39.2\,\mu = 3.2$, so $\mu = 0.5$
(b) $v = at = 0.8 \times 5 = 4$; after removal deceleration = $\mu g = 4.9$
$0 = 4^2 - 2 \times 4.9 \times s$, $s = 1.63$

Answer: (a) $\mu = 0.5$ (b) 1.63 m

4 Double angle equations (All boards)

Solve $\cos 2\theta - 3\cos\theta = -2$
for $0° \le \theta \le 360°$.

$\cos 2\theta \equiv 2\cos^2\theta - 1$: $2\cos^2\theta - 3\cos\theta + 1 = 0$

Solve the quadratic: $\cos\theta = 1$ or $\cos\theta = \frac{1}{2}$

$\cos\theta = 1$: $\theta = 0°, 360°$

$\cos\theta = \frac{1}{2}$: $\theta = 60°, 300°$

Answer: $\theta = 0°, 60°, 300°, 360°$

5 Cubic inequalities (All boards)

Given that $(x - 4)$ is a factor of $f(x) = x^3 + x^2 - 14x - 24$, solve $f(x) \le 0$.

Factorise: $f(x) = (x + 3)(x + 2)(x - 4)$
Roots at $x = -3, -2, 4$
Sketch the cubic (positive x^3 coefficient: negative, positive, negative, positive)
Read off where $f(x) \le 0$.

Answer: $x \le -3$ or $-2 \le x \le 4$

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6 Parametric differentiation: tangent and normal (All boards)

A curve has parametric equations $x = t^2 + 1$, $y = t^3 - t$.

Find $\frac{dy}{dx}$ in terms of t, and the equation of the normal at the point where $t = -1$, in the form $ax + by + c = 0$ (integers).

$\frac{dy}{dx} = \frac{dy}{dt \div dx/dt} = \frac{3t^2 - 1}{2t}$

$t = -1$: point (2, 0), gradient -1

Normal gradient $= -1 \div (-1) = 1$, so normal: $x - y - 2 = 0$

Answer: $\mathbf{\frac{dy}{dx} = \frac{3t^2 - 1}{2t}}$**; x − y − 2 = 0**

7 Binomial expansion: finding n and k (All boards)

In the expansion of $(1 + kx)^n$, where n is a positive integer, the coefficient of x is 20 and the coefficient of x^2 is 180. Find n and k.

$nk = 20$ and $\frac{n(n-1)}{2}k^2 = 180$

$(n - 1)k = \frac{2 \times 180}{20} = 18$ (dividing the second equation by the first)

$n\,k - (n - 1)\,k = k = 20 - 18 = 2$

$n = \frac{20}{2} = 10$

Answer: n = 10, k = 2

8 Continuous random variable: cumulative distribution and median (CIE)

The random variable X has pdf $f(x) = \frac{4}{256}x^3$ for $0 \le x \le 4$, and 0 otherwise.

(a) Find the cumulative distribution function F(x) for $0 \le x \le 4$.
(b) Find P(X > 2). (c) Find the median of X to 3 s.f.

(a) $F(x) = \int_0^x f(t)\,dt = \frac{x^4}{256}$

(b) $P(X > 2) = 1 - F(2) = \frac{15}{16}$

(c) $F(m) = \frac{1}{2}$: $m^4 = \frac{4^4}{2}$, so $m = 4 \times (0.5)^{\frac{1}{4}}$

$m = 3.36$

Answer: (b) $\mathbf{\frac{15}{16}}$ **(c) 3.36**

9 Differentiating sec, cosec, tan and cot (All boards)

$y = x\,\mathrm{cosec}(3x)$.

Find the exact gradient at $x = \frac{\pi}{9}$.

Product rule: $\frac{dy}{dx} = \mathrm{cosec}(3x) + x(-3\mathrm{cosec}(3x)\cot(3x))$

At $x = \frac{\pi}{9}$: $\mathrm{cosec}(3x) = \frac{2\sqrt{3}}{3}$ and the derivative term $= -2$

Gradient $= \frac{2\sqrt{3}}{3} + \frac{\pi}{9} \times (-2) = \frac{6\sqrt{3} - 2\pi}{9}$

Answer: gradient $\mathbf{\frac{6\sqrt{3} - 2\pi}{9}}$

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10 Partial fractions and binomial expansion (All boards)

$f(x) = \frac{-18x + 2}{(1 - 3x)(1 + 3x)}$.

(a) Express f(x) in partial fractions.

(b) Hence expand f(x) up to x^2.

(c) State the range of x for which the expansion is valid.

(a) $f(x) = \frac{A}{(1 - 3x)} + \frac{B}{(1 + 3x)}$: $A = -2$, $B = 4$

(b) $-2(1 - 3x)^{-1} = -2(1 + 3x + 9x^2 ...)$

Add the series for both fractions.

(c) Need $|3x| < 1$ and $|3x| < 1$

Answer: (a) $\frac{-2}{(1 - 3x)} + \frac{4}{(1 + 3x)}$ **(b)** $2 - 18x + 18x^2$ **(c)** $|x| < \frac{1}{3}$