Day 315 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Transformations from a graph (All boards)

The diagram shows the curve y = f(x), with turning points A and B.
Find the coordinates of the turning points of
(a) y = f(x) − 1 (b) y = −f(x)

(a) y = f(x) − 1: A → (−4, 6), B → (−2, −2)
(b) y = −f(x): A → (−4, −7), B → (−2, 1)
Answer: (a) (−4, 6), (−2, −2); (b) (−4, −7), (−2, 1)

2 Projectile from and to the ground (AQA, Edexcel, OCR, MEI, Eduqas)

A ball is projected from a point O on horizontal ground at 10 m s^{-1} at angle α above the horizontal, $\tan\alpha = \frac{12}{5}$, and lands at A. Find (a) the time of flight, (b) the greatest height, (c) OA. Take g = 9.8. Give answers to 3 s.f. if not exact.

Horizontal component = $10\times\frac{5}{13} = \frac{50}{13}$; vertical component = $10\times\frac{12}{13} = \frac{120}{13}$

(a) Vertical: $0 = \frac{120}{13}t - 4.9t^2$, so $t = \frac{1200}{637}$

(b) At the top: $0 = \left(\frac{120}{13}\right)^2 - 2\times9.8\times H$, so $H = \frac{36000}{8281}$

(c) OA = $\frac{50}{13}\times\frac{1200}{637}$

Answer: (a) 1.88 s (b) 4.35 m (c) 7.25 m

3 Change of sign to locate a root (All boards)

f(x) = ln x + 5x − 12.
Show that the equation f(x) = 0 has a root α in the interval [2, 3].

f(x) is continuous on the interval
f(2) = −1.31 < 0
f(3) = 4.10 > 0
A change of sign, so a root lies in [2, 3]
Answer: Sign change: f(2) = −1.31, f(3) = 4.10

4 Product moment correlation coefficient (AQA, Edexcel, OCR, MEI, Eduqas)

For n = 9 pairs of (x, y): Σx = 100, Σy = 439,
$\Sigma x^2 = 1440$, $\Sigma y^2 = 21877$, Σxy = 5178.
Calculate the product moment correlation coefficient, and describe the correlation.

$S_{xx} = 1440 - \frac{100^2}{9} = 329$

$S_{yy} = 21877 - \frac{439^2}{9} = 464$

$S_{xy} = 5178 - \frac{100\times439}{9} = 300$

$r = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}} = 0.769$

This is strong positive correlation
Answer: r = 0.769, positive correlation

5 Factor theorem: roots with surds (All boards)

$f(x) = x^3 + 4x^2 + 4x + 1$.
(a) Show that (x + 1) is a factor of f(x).
(b) Solve f(x) = 0, giving surds in simplest form.

f(−1) = −1 + 4 − 4 + 1 = 0, so (x + 1) is a factor.
$f(x) = (x + 1)(x^2 + 3x + 1)$

$x^2 + 3x + 1 = 0$ gives $x = \frac{-3\pm\sqrt{5}}{2}$

Answer: x = −1 or $x = \frac{-3\pm\sqrt{5}}{2}$

6 Angle between vectors (3D) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The points A, B and C have coordinates (2, 4, −3), (2, −2, −3) and (1, 0, −3).
Find angle ABC, in degrees to 1 d.p.

BA = $\begin{pmatrix}0\\6\\0\end{pmatrix}$, BC = $\begin{pmatrix}-1\\2\\0\end{pmatrix}$

BA·BC = 12

|BA| = $\sqrt{36}$, |BC| = $\sqrt{5}$

cos ABC = $\frac{12}{\sqrt{36}\times\sqrt{5}}$ = 0.8944, so angle ABC = 26.6°

Answer: 26.6°

7 Smooth incline: passing a point twice (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle is projected up a line of greatest slope of a smooth plane inclined at α. It passes A, 7.35 m from the start, after 1 s and again after 3 s. Find (a) the speed of projection, (b) α to 1 d.p. Take g = 9.8.

Deceleration a = g sin α (unknown). s = ut − ½at² at both times:
u×1 − ½a×1² = u×3 − ½a×3² = 7.35
Subtracting: u(3 − 1) = ½a(9 − 1), so u = ½a(4)
Substituting: ½a(4)(1) − ½a×1² = 7.35, so ½a×3 = 7.35, a = 4.9
(a) u = 2.45×4 = 9.8
(b) 9.8 sin α = 4.9, sin α = 0.5, α = 30.0°

Answer: (a) 9.8 m s^{-1} (b) 30.0°

8 Binomial in two stages (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Parts are defective with probability 0.3, independently. A box of 8 parts is rejected if it has at least 2 defective.
From 6 such boxes, find the probability that exactly 3 are rejected (3 s.f.).

Y ~ B(8, 0.3). P(rejected) = P(Y ≥ 2) = 1 − 0.255 = 0.745
W ~ B(6, 0.745) is the number rejected
P(W = 3) = C(6, 3)(0.745)^3(1 − 0.745)^3 = 0.137

Answer: 0.137

9 Partial fractions and binomial expansion (All boards)

f(x) = $\frac{-13x+3}{(1-3x)(1+x)}$.

(a) Express f(x) in partial fractions.
(b) Hence expand f(x) up to x².
(c) State the range of x for which the expansion is valid.

(a) f(x) = $\frac{A}{(1-3x)}+\frac{B}{(1+x)}$: A = −1, B = 4
(b) −1(1 − 3x)$^{-1}$ = −1(1 + 3x + 9x² ...)
Add the series for both fractions.
(c) Need |3x| < 1 and |x| < 1

Answer: (a) $\frac{-1}{(1-3x)}+\frac{4}{(1+x)}$ (b) 3 − 7x − 5x² (c) |x| < $\frac{1}{3}$

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10 Perpendicular vectors and a constant (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The vectors $\begin{pmatrix} k \\ k \\ -1 \end{pmatrix}$ and $\begin{pmatrix} k \\ 4 \\ 5 \end{pmatrix}$ are perpendicular.

Find the possible values of k and, for the larger value, the exact length of the first vector.

Scalar product = 0: $k^2 + 4k - 5 = 0$

$(k - 1)(k + 5) = 0$

$k = -5$ or $k = 1$. For $k = 1$ the first vector is $\begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$, with length $\sqrt{3} = \sqrt{3}$

Answer: k = −5 or 1; length $\sqrt{3}$